Horses and Rabbits? Trade-Off Theory and Optimal Capital Structure

Appendix B

In this appendix we develop a dynamic counterpart to the static Leland (1994) model. In this model the firm optimally issues a perpetual callable bond with coupon rate C and principal P at time zero. This debt issue ceases to exist under either of two conditions. First, if the firm value $V(t)$ reaches a lower boundary V_B (which is an endogenously determined bankruptcy level) before an upper level V_U (which is a restructuring level chosen to maximize the initial total levered firm value), the firm goes bankrupt and the debt holders receive $(1-\alpha_{BC})V_B$ (where α_{BC} is the bankruptcy cost parameter). Second, if $V(t)$ reaches the upper boundary V_U before the lower level V_B, the debt is called at face value. At the same time, a new issue of another perpetual callable bond is optimally issued. Since the time when either boundary is first hit is random, the price of the debt at any point before either boundary is hit does not depend explicitly on time. It satisfies the following ordinary differential equation (ODE),

$$\frac{1}{2}\sigma^2 V^2 D_{VV} + (r-\delta)VD_V - rD + C = 0. \tag{B.1}$$

The solution to this ODE is

$$D(V) = \frac{C}{r} + aV^{x_1} + bV^{x_2}, \tag{B.2}$$

where

$$\begin{aligned} x_1 &= \frac{-\left(r-\delta-\sigma^2/2\right) + \sqrt{\left(r-\delta-\sigma^2/2\right)^2 + 2\sigma^2 r}}{\sigma^2}, \\ x_2 &= \frac{-\left(r-\delta-\sigma^2/2\right) + \sqrt{\left(r-\delta-\sigma^2/2\right)^2 + 2\sigma^2 r}}{\sigma^2}, \end{aligned} \tag{B.3}$$

and a and b are determined by boundary conditions. Applying the two boundary conditions,

$$D(V_B) = (1-\alpha)V_B, \qquad D(V_U) = P, \tag{B.4}$$

we have

$$a = \left(V_U^{x_2}\left((1-\alpha)V_B - C/r\right) - V_B^{x_2}\left(P - C/r\right)\right)/\Sigma,$$
$$b = \left(V_B^{x_2}\left(P - C/r\right) - V_U^{x_2}\left((1-\alpha)V_B - C/r\right)\right)/\Sigma, \tag{B.5}$$

where $\Sigma = V_B^{x_1}V_U^{x_2} - V_U^{x_1}V_B^{x_2}$. As is typical in practice, we require the debt to be issued at par. Setting $D(V(0)) = P$, and solving for P, we have

$$D(V) = P = \frac{C}{r} + \left((1-\alpha_{BC})V_B - \frac{C}{r}\right)\frac{V_U^{x_2}V(0)^{x_1} - V_U^{x_1}V(0)^{x_2}}{V_U^{x_2}V_B^{x_1} - V_U^{x_1}V_B^{x_2} + V_B^{x_2}V(0)^{x_1} - V_B^{x_1}V(0)^{x_2}}. \tag{B.6}$$

Let $F(V)$ be the price of a contingent claim which in addition to a payoff at the boundaries, has claim to a cashflow E. Then $F(V)$ has the following solution,

$$F(V) = E/r + c_1V^{x_1} + c_2V^{x_2}, \tag{B.7}$$

where c_1 and c_2 are determined by the boundary conditions.

If we let $TB(V)$ represent the total tax shields (from current outstanding debt and all future debt issues), then the cashflow is $E = \tau\ C$, $TB(V_B) = 0$, and $TB(V_U) = TB(V)(V_U/V(0))$. The last condition results from the scaling property discussed in Appendix A because the total tax shields at the restructuring point V_U should be $(V_U/V(0))$ as large as it is at $V(0)$. Now, a straightforward derivation yields the following solution for $TB(V)$,

$$TB(V) = \tau C/r + hV^{x_1} + kV^{x_2}, \tag{B.8}$$

where

$$h = \left(-V_U^{x_2}\tau C/r - V_B^{x_2}\left(TB(V(0))V_U/V(0) - \tau C/r\right)\right)/\Sigma,$$
$$k = \left(-V_U^{x_1}\tau C/r - V_B^{x_1}\left(TB(V(0))V_U/V(0) - \tau C/r\right)\right)/\Sigma, \tag{B.9}$$

and $TB(V(0)) = \dfrac{tb(V(0))}{1-\phi}$, where

$$tb(V(0)) = \tau C/r \left(1 + \frac{\left(V(0)^{x_1} V_B^{x_2} - V_B^{x_1} V(0)^{x_2}\right) + \left(V_U^{x_1} V(0)^{x_2} - V(0)^{x_1} V_U^{x_2}\right)}{V_B^{x_1} V_U^{x_2} - V_U^{x_1} V_B^{x_2}} \right), \tag{B.10}$$

$$\phi = \frac{\left(V_B^{x_2} V(0)^{x_1} - V_B^{x_1} V(0)^{x_2}\right) V_U}{\left(V_B^{x_2} V_U^{x_1} - V_B^{x_1} V_U^{x_2}\right) V(0)}. \tag{B.11}$$

Similarly, the total bankruptcy costs are given by

$$BC(V) = fV^{x_1} + wV^{x_2}, \tag{B.12}$$

where

$$\begin{aligned} f &= \left(V_U^{x_2} \alpha_{BC} V_B - V_B^{x_2} BC(V(0)) V_U / V(0)\right) / \Sigma, \\ w &= \left(V_B^{x_1} BC(V(0)) V_U / V(0) - V_U^{x_1} \alpha_{BC} V_B\right) / \Sigma, \end{aligned} \tag{B.13}$$

and $BC(V(0)) = \dfrac{bc(V(0))}{1-\phi}$, where

$$bc(V(0)) = \alpha_{BC} V_B \left(\frac{V_U^{x_1} V(0)^{x_2} - V(0)^{x_1} V_U^{x_2}}{V_U^{x_1} V_B^{x_2} - V_B^{x_1} V_U^{x_2}} \right). \tag{B.14}$$

To prevent the firm from restructuring continuously, we introduce a proportional transaction cost for issuing new debt. The total transaction cost is

$$TC(V) = qV^{x_1} + sV^{x_2}, \tag{B.15}$$

where

$$q = -\frac{TC(V(0)) V_U V_B^{x_2}}{V(0)\Sigma}, \qquad s = \frac{TC(V(0)) V_U V_B^{x_1}}{V(0)\Sigma}, \tag{B.16}$$

and $TC(V(0)) = \dfrac{tc(V(0))}{1-\phi}$, $tc(V(0)) = \beta P$, where β is the transaction cost on each dollar issued and P is the principal of the initial debt.

The total value of the levered firm is the value of the unlevered firm V, plus the total tax shields $TB(V)$, less the total bankruptcy costs $BC(V)$, less the total transaction cost $TC(V)$,

$$TLV(V) = V + TB(V) - BC(V) - TC(V) . \tag{B.17}$$

The equity value after the debt is issued is

$$E(V) = TLV(V) - D(V) = V + (\tau - 1)C/r + (h - f - q - a)V^{x_1} + (k - w - s - b)V^{x_2} . \tag{B.18}$$

The limited liability of the equity is satisfied if $\partial E(V) / \partial V \,|_{V=V_B} = 0$ (which is a smooth-pasting condition). That is,

$$1 + x_1(h - f - q - a)V_B^{x_1 - 1} + x_2(k - w - s - b)V_B^{x_2 - 1} = 0 . \tag{B.19}$$

Therefore, the firm maximizes $TLV(V(0))$, subject to the condition in equation (B.19).