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DOES PRIOR PERFORMANCE AFFECT A MUTUAL FUND'S CHOICE OF RISK? THEORY AND FURTHER EMPIRICAL EVIDENCE*

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Abstract

Recent empirical studies of mutual fund competition examine the relation between a fund's performance, the fund manager's compensation, and the fund manager's choice of portfolio risk. This paper models a manager's portfolio choice for compensation rules that can be either a concave, linear, or convex function of the fund's performance relative to that of a benchmark. For particular compensation structures, a manager increases the fund's "tracking error" volatility as its relative performance declines. However, declining performance does not necessarily lead the manager to raise the volatility of the fund's return.

The paper performs non-parametric and parametric tests of the relation between mutual fund performance and risk-taking for more than 6,000 equity mutual funds over the 1962 to 2006 period. There is a tendency for mutual funds to increase the standard deviation of tracking errors, but not the standard deviations of returns, as their performance declines. This risk-shifting behavior appears more common for funds whose managers have longer tenures.

I. Introduction

As mutual fund investing has grown, the management of mutual funds has come under closer scrutiny by financial economists. One strand of research examines potential agency problems between a mutual fund's shareholders and its portfolio manager. Several studies investigate whether a manager might unnecessarily shift the fund's risk in response to changes in its performance relative to other funds. This behavior is linked to the way the manager is compensated and to the actions of mutual fund investors. The manager's compensation depends on her success in generating flows of new investments into the fund, while mutual fund investors "chase returns" by channeling investments into funds with better relative performance. This creates a situation described as a mutual fund "tournament" where portfolio managers compete for better performance, greater fund inflows, and, ultimately, higher compensation.

Inflows rise nonlinearly with a fund's relative performance. Numerous studies document that mutual funds with the best recent performance experience the lion's share of new inflows, but poorly performing funds are not penalized with sharply higher outflows.[1] If the fund manager's compensation rises in proportion to the fund's inflows, this convex performance - fund flow relation produces a convex performance - compensation structure.[2] Research, such as Sirri and Tufano (1998), notes that such compensation is similar to a call option, creating an incentive for a manager to raise the risk of the fund's relative returns. Chevalier and Ellison (1997), Brown, Harlow, and Starks (1996), Busse (2001), and Goriaev, Nijman, and Werker (2005) have

[1] Studies examining the fund flow – performance relationship include Ippolito (1992), Gruber (1996), Chevalier and Ellison (1997), Sirri and Tufano (1998), Goetzmann and Peles (1997), and Del Guercio and Tkac (2002).

[2] The literature on mutual fund tournaments distinguishes between a fund's investment advisor, the entity responsible for portfolio management, and the portfolio manager hired by the advisor. The typical investment advisor is paid a fixed fraction of the fund's assets, assets that depend on both net fund inflows (external growth of assets) and the fund's return (internal growth of assets). However, the portfolio manager's compensation is assumed to depend only on his ability to generate extraordinary growth in fund assets, growth that depends on the fund's return relative to (the average of) other funds' returns. Common or systematic shocks to all funds' returns (affecting internal asset growth) are not due to the individual manager's portfolio selection ability and would not affect compensation. Hence, compensation is assumed to depend on relative, not absolute, performance.

empirically examined the behavior of a cross-section of mutual funds for which this risk-taking incentive is predicted to differ.

The current paper adds to this mutual fund tournament literature by providing new theoretical and empirical insights into risk-taking by mutual funds. It models the optimal intertemporal portfolio strategy of a mutual fund manager that faces the competitive tournament environment assumed by recent empirical work. Explicit solutions for this manager's portfolio allocation are derived when her utility displays constant relative risk aversion and compensation is either a concave, linear, or convex function of the fund's relative calendar-year performance.

The model shows that the deviation of a fund manager's optimal portfolio from the benchmark portfolio is a function of the fund's performance. If the penalty for poor performance is limited so that the manager's total compensation can never fall to zero, then the fund manager chooses to deviate more from the benchmark portfolio as the fund's relative performance declines. In other words, when a fund is performing poorly it displays more "tracking error" than when it performs relatively well. However, it is not necessarily true that an under-performing fund chooses to raise the volatility (standard deviation) of its returns.

Almost all empirical studies that have tested for tournament risk-shifting have analyzed fund risk measures other than tracking error. Most commonly, empirical research has tested for whether underperforming funds increase the standard deviation of their total returns, rather than the standard deviation of their tracking errors. The most comprehensive of these studies conclude that there is no evidence for tournament behavior. However, based on our model's insights, we argue that this conclusion may be unwarranted because it is based on tests that have employed risk measures that are inappropriate for examining the tournament hypothesis.

This paper re-examines the empirical evidence for tournament behavior in light of our model's predictions. We construct two different types of tests. One is a non-parametric test that modifies the standard deviation ratio (SDR) tests used in prior studies. It analyzes risk-shifting for a cross-section of mutual funds based on the SDR of their tracking errors, rather than the SDR of their returns. Another parametric test examines individual mutual funds' time series behavior based on an empirical model that nests our paper's theoretical one. Unlike the non-parametric tests that allow a fund's risk to change only once per year, this parametric test permits each fund's risk to vary at every (monthly) observation date.

Both tests are performed using data on more than 6,000 mutual funds that operated during the 1962 to 2006 period. As predicted by our model, the empirical evidence is supportive of behavior where an under-performing mutual fund manager increases the standard deviation of tracking error, but not the standard deviation of returns. Evidence is strongest for managers having longer tenures at their funds, a result that also is consistent with our model.

The plan of the paper is as follows. Section II briefly discusses related theoretical and empirical work on the risk-taking incentives of mutual fund managers. In Section III we present our model. Section IV discusses non-parametric and parametric empirical methods for testing the risk-taking behavior implied by our model. Section V describes our data, while Section VI presents the empirical results. Concluding comments are in Section VII.

II. Related Literature

This section begins by discussing some of the theoretical research that relates to our paper's model. It then reviews empirical studies of mutual fund tournaments.

A. Models of Portfolio Management

A growing literature examines links between a fund manager's compensation contract and his portfolio choice. Grinblatt and Titman (1989) show how compensation contracts that include a bonus for good performance can produce moral hazard incentives. Mutual fund managers can maximize the present value of their option-like bonus by choosing a fund portfolio with excessive risk. Moreover, the fund manager can risklessly capture the increased value of this bonus if she could hedge using her personal wealth.

Starks (1987) considers the moral hazard incentives of a bonus contract, focusing on situations of asymmetric information between investors and fund managers. When investors cannot observe a manager's choice of portfolio risk or the manager's effort level, compensation contracts with symmetric payoffs dominate contracts that include a bonus. However, Das and Sundaram (2002) show that the relative advantages of symmetric and bonus contracts can be reversed if investors' choice of funds is made endogenous to the funds' risk levels and compensation contracts. In their model, bonus contracts provide better risk-sharing between investors and fund managers when investors take account of a fund's risk and contract choice. Dybvig, Farnsworth, and Carpenter (2002) also find that contracts should include a bonus proportional to the fund's return in excess of a benchmark return when a fund manager's effort determines the quality of her information.[3]

Other research, such as Huberman and Kandel (1993), Heinkel and Stoughton (1994), and Huddart (1999), considers environments where fund managers possess different abilities that are unbeknownst to investors. In these screening models, there is typically an initial period when investors learn of managers' abilities based on their relative performances, followed by a second period when investors can switch their savings to those managers perceived to have the highest abilities. Hence, these "investor learning" models can explain the link between fund flows and prior performance. In addition, if managerial ability displays decreasing returns to scale, Berk and Green (2004) show that fund flows determine the relative sizes of mutual funds such that, in equilibrium, investors expect no future superior returns net of fund fees and expenses.

The model in the current paper differs from this previous work by focusing on how prior performance affects a fund manager's intertemporal choice of portfolio risk. We take the structure of compensation as given and study a manager's dynamic portfolio choice during an annual mutual fund tournament. Models by Carpenter (2000), Cuoco and Kaniel (2001), and

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[3] Becker, Ferson, Myers, and Schill (1999) also analyze the consequences for portfolio choice when a manager has compensation based on the fund's return in excess of a benchmark and also has the ability to

Basak, Pavlova, and Shapiro (2007) are related to ours. These papers assume that the performance-related components of a manager's compensation are piece-wise linear functions of performance. Carpenter (2000) specifies compensation equal to a fixed-fee plus a call option written on the value of the managed portfolio with an exercise price equal to a benchmark asset. Cuoco and Kaniel (2001) permit compensation to contain a penalty for poor performance in the form of the manager writing a put option on the managed portfolio. Basak, Pavlova, and Shapiro (2007) assume a manager's compensation is linear in portfolio returns relative to a benchmark but subject to fixed minimum and maximum payoffs, equivalent to a bull spread option strategy.

While we also assume that a manager's compensation depends on the portfolio's performance relative to a benchmark, the contract is not strictly in the form of standard call or put options. Rather than being an option-like, piece-wise linear function of performance, our compensation contract is a smooth function that can be concave, linear, or convex in relative performance. An advantage of our smooth compensation schedule is that it leads to simple and intuitive closed-form solutions for a fund manager's optimal portfolio choice. This simplicity allows the model to guide our later empirical tests of mutual fund behavior.

Arguably, a smooth performance – compensation function better captures the environment of a mutual fund tournament where compensation is proportional to a fund's assets under management that, in turn, depend on how investor inflows respond to the fund's relative performance.[4] Chevalier and Ellison ((1997), p.1181) contend that assuming a smooth relationship between mutual fund performance and fund flows is preferable because it avoids imposing the strong restrictions on risk incentives that occur with a piece-wise linear contract. Under a piece-wise linear contract, a manager's risk incentives are always maximized or

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time the market.

[4] The contract in Carpenter (2000) may better represent the compensation of a non-financial firm manager who receives stock options. Cuoco and Kaniel's (2001) compensation contract might be most appropriate for other portfolio managers, such as managers of pension funds. Their analysis focuses on the equilibrium asset pricing consequences of portfolio management. Basak, Pavlova, and Shapiro (2007) assume a

minimized at the contract's kink points, yet for mutual funds identifying the location of these kink points is subject to error since they must be estimated from past fund flows.

B. Empirical Research on Mutual Fund Tournaments

Several empirical studies have analyzed the relationship between a mutual fund's prior performance and its choice of risk. Chevalier and Ellison (1997) estimate the shape of mutual funds' performance - fund flow relation and use it to infer different funds' risk-taking incentives. They, like other studies, assume that a fund's inflows respond primarily to its relative performance calculated over the previous calendar year. Thus fund managers compete in annual tournaments that begin in January and end in December.[5] A fund's risk-taking incentive over the final quarter of the year is assumed to be proportional to the estimated convexity of fund inflows measured locally around the fund's September performance ranking. Using 1983 to 1993 data on the equity holdings of mutual funds at the ends of September and December, they find that a fund tends to change the standard deviation of its return relative to a benchmark return as the performance - fund flow relation predicts. For example, young mutual funds that perform relatively poorly from January to September tend to raise the standard deviation of their return in excess of a benchmark return (standard deviation of tracking error) during October to December.[6]

Another study by Brown, Harlow, and Starks (1996), hereafter referred to as BHS, performs standard deviation ratio (SDR) tests of whether a fund that performs relatively poorly at mid-year tends to raise the standard deviation of its return over the latter half of the year more than does a fund that performs relatively well at mid-year. They use monthly returns data for a

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complete markets environment where portfolio managers lack asset selection ability and choose only systematic risk.

[5] This assumption is justified because sources of mutual fund information, such as Morningstar, Inc., typically compute relative fund performances using this calendar year period. Hence, flows of investor funds and, in turn, managerial compensation should be most sensitive to a fund's calendar year performance. Empirical evidence in Koski and Pontiff (1999) indicates that changes in a fund's risk are most strongly related to performance calculated over calendar years.

[6] Chavalier and Ellison (1997) also test their estimated performance - flow relation using monthly data on fund returns. They measure a fund's risk as the standard deviation of its return in excess of the return on a

cross-section of mutual funds during 1980 to 1991 and find support for the tournament hypothesis that mid-year "losers" gamble to improve their relative end-of-year performance by raising their funds' standard deviation of returns more than do mid-year "winners." Koski and Pontiff (1999) also use monthly returns data to calculate over the 1992 to 1994 period various measures of a fund's risk, including the standard deviation, beta, and idiosyncratic risk of a fund's returns. Their results are similar to those of BHS (1996) in that a mutual fund's performance in the first half of the calendar year is negatively related to its change in risk during the second half.

However, more recent research finds that some of these results are not robust to other testing methods and sample periods. Busse (2001) uses a different database of daily, rather than monthly, mutual fund returns from 1985 to 1995 to calculate more accurate estimates of a fund's standard deviation of returns. He duplicates the SDR tests in BHS (1996) and finds no evidence that mid-year poor-performing funds increase their standard deviation of return more than mid-year better-performing funds. He also shows that if standard deviations are calculated using monthly returns measured from the middle of each month, rather than from the beginning of each month as in BHS (1996), the evidence for raising return standard deviations as relative performance declines disappears.

Similarly, Goriaev, Nijman, and Werker (2005), hereafter referred to as GNW, replicate BHS's SDR tests using an expanded 1976 to 2001 sample of monthly fund returns and correct their test significance levels for cross-correlation in fund returns. They also find no evidence that underperforming funds raise their standard deviations of returns. Hence, the more comprehensive studies of Busse (2001) and GNW (2005) conclude that the BHS (1996) finding of tournament behavior is fragile. It does not hold up to more precise tests and larger samples of mutual fund returns.

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value-weighted index of NYSE, AMEX, and NASDAQ stocks and find that it moves in the predicted direction during the last quarter of the year.

As our model of mutual fund tournaments in the next section demonstrates, under plausible conditions, an optimizing manager chooses to raise the standard deviation of her fund's tracking error (return in excess of a benchmark) as the fund's relative performance declines. Such behavior does not necessarily imply a rise in the fund's standard deviation of returns, beta, or residual risk. Hence, with the exception of Chevalier and Ellison (1997), prior tests of tournament behavior are based on arguably inappropriate risk measures. In particular, Busse's (2001) and GNW's (2005) rejection of tournament behavior may be unjustified since their SDR tests, like those of BHS (1996), employ standard deviations of total returns, rather than tracking error. Re-examining the evidence for tournament behavior with a focus on mutual funds' tracking errors, rather than their total returns, motives our paper's empirical work.

III. Modeling a Mutual Fund Manager's Portfolio Decisions

We now describe our model's specific assumptions. A fund manager's compensation is assumed to depend on the fund's performance relative to a benchmark index. The fund's portfolio can be invested partly in this benchmark index and partly in a set of "alternative" securities chosen by its fund manager. These alternative securities are defined as the portion of the fund's total assets that accounts for the difference between the fund's portfolio and one that is invested solely in the benchmark portfolio. The Appendix shows that when securities' excess returns and return covariances are constant, the fund manager's optimal choice of individual alternative securities is one where their relative portfolio proportions do not vary over time. This implies that the manager's intertemporal portfolio choice problem can be transformed to one of allocating a portion of the fund's portfolio to the benchmark index and the remaining portion to a single alternative composite security.[7] Hence, we simplify the presentation by assuming at the start that the portfolio allocation problem involves only two types of securities: the benchmark index and a single alternative security.

[7] This "two-fund separation" result is similar to that of Merton's (1971) case of lognormal asset prices.

Define S_t as the value of the relevant benchmark index at date t and A_t as the date t value of the alternative securities. S_t and A_t are assumed to follow the processes

(1) $$dS/S = \alpha_S dt + \sigma_S dz$$

(2) $$dA/A = \alpha_A dt + \sigma_A dq$$

where $\sigma_S dz \sigma_A dq = \sigma_{AS} dt$. For analytical convenience, σ_A, σ_S, and σ_{AS} are assumed to be constants. α_A and α_S may be time varying, as might be the case if market interest rates are stochastic.[8] However, we require that the spread between their expected rates of return, $\alpha_A - \alpha_S$, be constant.

If the fund manager allocates a portfolio proportion of $1-\omega$ to the benchmark index and a proportion ω to the alternative securities, then the portfolio's value, V, follows the process:

(3) $$\begin{aligned} dV/V &= (1-\omega)\,dS/S + \omega\, dA/A \\ &= \left[(1-\omega)\alpha_S + \omega\alpha_A\right]dt + (1-\omega)\sigma_S dz + \omega\sigma_A dq \end{aligned}$$

Note that whenever $\omega \neq 0$, the fund's return in (3) deviates from the benchmark return. We can also calculate the process followed by the fund's relative performance. Define $G_t \equiv V_t/S_t$ to be the date t ratio of the value of a share of the fund's portfolio to that of the benchmark. A simple application of Itô's lemma shows that

(4) $$dG/G = \omega\left(\alpha_A - \alpha_S + \sigma_S^2 - \sigma_{AS}\right)dt + \omega\left(\sigma_A dq - \sigma_S dz\right)$$

The fund manager is assumed to compete in a tournament for inflows into the fund. At the start of the tournament's assessment period, $G = 1$ by definition, but then changes stochastically according to equation (4). Thus, G_t measures the date t ratio of the fund's return to that of the benchmark since the start of the tournament, and hence $D_t \equiv \ln(G_t)$ is the difference between the fund's continuously-compounded return and that of the benchmark index since the

[8] The Appendix derives the values of α_A and σ_A in terms of the parameters of processes for n individual alternative securities.

beginning of the tournament.[9] The tournament ends at date T, which, for example, could be the last trading day of the calendar year. The manager's compensation is a function of the fund's relative performance at the end of the tournament, so that his compensation or "pay" can be written as $P[G_T]$.[10] [11]

The fund manager maximizes his expected utility of compensation (wealth) at the end of the tournament by choosing the fund's asset allocation at each point in time during the assessment period.[12] This maximization problem can be written as

(5) $$\underset{\omega(s)\forall s\in[t,T]}{\text{Max}} E_t\left[U\left(P\left[G_T\right]\right)\right]$$

subject to the process followed by G given in equation (4). If we define $J(G, t)$ as the derived utility of wealth (or performance) function, then assuming that $U(P[G_T])$ is concave in G_T, the first order condition of the Bellman equation with respect to ω implies that the portfolio proportion invested in the alternative securities is

(6) $$\omega^* = -\frac{J_G \alpha_G}{G J_{GG} \sigma_G^2}$$

[9] We refer to relative performance as the ratio of returns, G_t, rather than the difference in returns, D_t, but this distinction is nonessential. The manager's compensation function could be rewritten in terms of $\ln G_t$, rather than G_t. As will be shown, a manager's optimal portfolio choice is independent of prior performance when compensation is proportional to a power of G_t rather than D_t, so the ratio is a natural variable to use.

[10] In practice, compensation may depend also on the performance of the overall equity (mutual fund) market. Karceski (2002) shows that a fund's inflows are highest when the fund performs relatively well and, simultaneously, the overall stock market performs well. He studies the implications of this phenomenon for fund managers' selection of high versus low beta stocks and the equilibrium effects on asset prices. Our analysis omits this market effect.

[11] The assumption that compensation depends only on the fund's performance relative to a single index is a simplification for another reason. In general, a fund's net inflows, and hence its manager's portfolio choices, might depend on the final performances of each of the mutual fund's competitors. Our simplified structure can be justified in an environment where there are a large (infinite) number of mutual funds that choose different "alternative" securities. Their relative performances over the year would be a smooth, approximately normally distributed, function around a mean performance. This would justify (as we do in our empirical work) using the "average" performance of all mutual funds as a sufficient statistic for comparing any given mutual fund's performance.

[12] Our model setting is very similar to that of Duffie and Richarson (1991) who study the trading strategy of a risk-averse hedger. We assume that the manager does not hedge his compensation risk via his personal portfolio. This is a standard assumption, though Grinblatt and Titman (1989) is an exception.

where $\alpha_G \equiv \alpha_A - \alpha_S + \sigma_S^2 - \sigma_{AS}$ and $\sigma_G^2 \equiv \sigma_A^2 - 2\sigma_{AS} + \sigma_S^2$. Substituting this back into the Bellman equation, one obtains an equilibrium partial differential equation for J which must satisfy the boundary condition $J(G_T,T) = U(P[G_T])$. A solution requires that the manager's utility function and compensation schedule be specified. We make the standard assumption that utility displays constant relative risk aversion, $U(P[G_T])=(P[G_T])^{\gamma}/\gamma$, where $\gamma < 1$. For the manager's compensation schedule, we choose a flexible specification that can be either a concave, linear, or convex function of fund performance:

$$P\left[G_T\right] = \left(a + \frac{b}{c} G_T\right)^c \tag{7}$$

where $b > 0$, $c > 0$, $a > - b/c$, and $c\gamma < 1$. When $0 < c < 1$, the manager's compensation is a concave function of performance, G_T. Compensation is linear in performance when $c = 1$, while when $c > 1$ the function is convex.[13] If a is set equal to 1 and one takes the limit of P as c goes to infinity, the function becomes exponential, $P[G_T] = \exp(bG_T)$. While most prior empirical studies of mutual funds emphasize the convexity of fund flows and compensation to performance, the allowance in equation (7) for a non-convex function may be useful for modeling money managers in other industries.[14]

As will be shown, the sign of the parameter a is critical to a manager's incentive to shift a fund's risk. In the linear case of $c = 1$, a can be interpreted as the fixed component of a manager's net compensation or end-of-period wealth. Plausible arguments can be made for a to be positive or negative, depending on the particular fund manager. For example, if a manager incurs fixed expenses (overhead) that are not explicitly reimbursed by the fund, then a could be

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[13] The function could be generalized to $P[G_T] = d(a + (b/c)G_T)^c$, where $d > 0$, but this extension has no effect on portfolio choice. If performance is defined as the difference in, rather than the ratio of, returns, then compensation is convex (concave) in $D_T \equiv \ln G_T$ whenever $a + bG_T$ is positive (negative). Since it will be shown that $a + (b/c)G_T$ is always positive in equilibrium, compensation can be a convex function of the difference in returns even when $0 < c \leq 1$. Thus, defining performance as the difference in returns expands the range for which compensation is convex in performance.

negative. On the other hand, if $P[G_T]$ is interpreted as a fund manager's total wealth that includes personal wealth as well as net compensation, then a may be positive if personal wealth is substantial. Personal wealth may tend to be greater for more experienced managers for at least two reasons. First, experienced managers are more likely to have savings from past compensation. Second, Chevalier and Ellison (1999) find that mutual fund managers that are older or have longer tenures at their funds are less likely to be terminated for underperformance; that is, they have more job stability. Hence, due to saved past compensation and the present value of future compensation, one might expect the parameter a to be greater for more experienced managers.

In general, when $c \neq 1$, the parameter a does not translate directly to a fixed component of wealth or total compensation, but its sign continues to determine whether total compensation has the potential to be non-positive. Lowering a (possibly below zero) decreases total compensation, but sensible solutions to the manager's portfolio choice problem require that $a > -b/c$. This restriction provides the manager with a feasible portfolio strategy that guarantees positive wealth at the end of the tournament. The manager can avoid zero wealth (and infinite marginal utility) by investing solely in the benchmark portfolio for the entire assessment period, since then compensation equals $(a + G_T b/c)^c = (a + b/c)^c > 0$.

The restriction $c\gamma < 1$ ensures that the boundary condition $U(P[G_T])=(P[G_T])^{\gamma}/\gamma$ is a concave function of G_T and that an interior solution to the manager's portfolio choice problem exists. This is always the case when $\gamma < 0$, that is, the manager's risk-aversion exceeds that of logarithmic utility. However, if $0 < \gamma < 1$ and c is sufficiently greater than 1 so that $\gamma c > 1$, then $U(P[G_T])$ is convex and the manager chooses ω to maximize the expected rate of return on G. From equation (4), this implies setting $\omega = +\infty$ if $\alpha_G > 0$, and $\omega = -\infty$ if $\alpha_G < 0$.

Assuming $c\gamma < 1$, the solution to the Bellman equation is

[14] Empirical evidence in Del Guercio and Tkac (2002) finds a performance - fund flow relation that

(8) $$J\left(G,t\right)=\frac{1}{\gamma}\left(a+\frac{b}{c}G\right)^{c\gamma}e^{-\theta(T-t)}$$

where $\theta\equiv -c\gamma\alpha_G^2/\left[2\left(1-c\gamma\right)\sigma_G^2\right]$. If equation (8) is substituted into equation (6), then the manager's optimal proportion invested in the alternative securities is

(9) $$\omega^*=\frac{\alpha_G}{\left(1-c\gamma\right)\sigma_G^2}\left(1+\frac{ac}{bG}\right)$$

Note from the restriction $a > - b/c$ that the term $\left(1+\frac{ac}{bG}\right)$ is always non-negative in equilibrium, even when $a < 0$. To see this, suppose that a is negative and that G declines sufficiently from its initial value of unity, so that $\left(1+\frac{ac}{bG}\right)$ approaches zero. Then from equation (9) the manager's optimal strategy is to invest fully in the benchmark portfolio. But at this point with $\omega^* = 0$, equation (4) implies $dG = 0$. With no further changes in the relative performance of the fund, the manager optimally prevents compensation from falling to zero.

Equation (9) shows that the manager chooses a long (short) position in the alternative securities whenever α_G is positive (negative), and the magnitude of the position is decreasing in risk aversion, $-\gamma$, but increasing in the fixed component of compensation, a.[15] Also, the manager's position is independent of the tournament's time horizon, $T-t$.[16] For the special case of $a = 0$, so that compensation is proportional to a power of relative performance, $P[G_T] = (bG_T/c)^c$, the alternative securities' portfolio weight is constant and invariant to changes in the fund's performance. However, for the general case of $a \neq 0$, ω^* varies with changes in G. When $a > 0$,

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appears linear for pension fund managers.

[15] If, as discussed earlier, the parameter a tends to be greater for more experienced fund managers, then equation (9) predicts that more experienced fund managers tend to deviate more from the benchmark portfolio, all else equal. Chevalier and Ellison's (1999) empirical results confirm this prediction. They find that older mutual fund managers tend to deviate more from the average portfolio chosen by other managers having the same investment style.

[16] That portfolio choice is independent of the investment horizon is a common feature of standard portfolio choice problems such as Merton (1971). The solution in (9) is analogous to that of a standard portfolio choice problem where the alternative securities portfolio plays the role of a risky asset portfolio and the benchmark portfolio is the risk-free asset. From this perspective, α_G and σ_G are the risky asset's excess

the manager moves closer to the benchmark portfolio with improvements in fund performance. The reverse occurs when $a < 0$. For the special case of $a = 1$ and $c \to \infty$, that is, compensation is the exponential form $P[G_T] = \exp(bG_T)$, equation (9) becomes

$$\omega^* = -\frac{\alpha_G}{\gamma b G \sigma_G^2} \tag{10}$$

so that the alternative securities' portfolio weight responds inversely to relative performance.

We summarize the manager's portfolio behavior with the following proposition.

***Proposition I*:** If a fund manager's utility displays constant relative risk aversion and has compensation given by (7), then when $c\gamma < 1$ an interior solution to the portfolio choice problem exists. Moreover, $\frac{\partial|\omega^*|}{\partial G} = -a\frac{c|\alpha_G|}{(1-c\gamma)bG^2\sigma_G^2}$, whose sign is opposite to that of the compensation parameter a. When a is positive (*negative*), a decline in the fund's relative performance leads the fund manager to deviate more (*less*) from the benchmark index.

The case $a > 0$ provides theoretical justification for Lakonishok, Shleifer, and Vishny's (1992) argument that successful managers attempt to lock-in gains. Furthermore, such behavior is likely to increase with the convexity of compensation, since when $\gamma < 0$ one can show that a larger value of c makes portfolio choice more sensitive to prior performance. In contrast, the $a < 0$ case leads to managerial risk-shifting behavior that is opposite to that assumed in recent empirical studies of tournaments. For this case, total compensation is not automatically bounded at zero, and a manager more closely matches the benchmark as performance declines to prevent zero compensation and infinite marginal utility.[17]

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return and its standard deviation of return, respectively, and risk aversion is $(1-c\gamma)bG/(ac+bG)$. Hence, the manager acts as if risk-aversion varies with performance.

[17] Proposition I is consistent with Carpenter (2000) and Cuoco and Kaniel (2001). When they assume that compensation equals a positive component plus a call option written on the portfolio's relative performance, they find that a manager increases tracking error as performance declines. This compares to our case of $a > 0$, since in both instances the compensation rule is always positive and portfolio managers

Proposition I has implications for the risk measures chosen by BHS (1996), Busse (2001), and GNW (2005). These studies test the standard deviation ratio (SDR) relation:

(11) $$\frac{\sigma_{2L}}{\sigma_{1L}} > \frac{\sigma_{2W}}{\sigma_{1W}} \quad .$$

where σ_{ij} denotes the standard deviation of the rate of return on mutual fund j's portfolio during the i^{th} half of the year. Mutual fund $j = L$ is a "loser" that displayed relatively poor performance in the first half of the year, while mutual fund $j = W$ is a "winner" that had relatively good performance during the first half. The implication is that a mutual fund that is a mid-year loser should increase the standard deviation of its fund more than a fund that was a mid-year winner.

Does our model imply the inequality in (11)? Note that the proportion invested in the alternative securities that would minimize the standard deviation of the mutual fund's rate of return given by equation (3) is

(12) $$\omega_{\min} = \frac{\sigma_S^2 - \sigma_{AS}}{\sigma_G^2}$$

Written in terms of (12), the optimal portfolio allocation in (9) becomes

(13) $$\omega^* = \omega_{\min} \frac{\alpha_G}{\left(\sigma_S^2 - \sigma_{AS}\right)\left(1 - c\gamma\right)} \left(1 + \frac{ac}{bG}\right)$$

This allows us to state the following proposition:

***Proposition II*:** Suppose $c\gamma < 1$ and $a > 0$, so that an interior solution exists in which the manager deviates more from the benchmark index as performance declines. If $\alpha_G / \left(\sigma_S^2 - \sigma_{AS}\right) > 0$ and $\frac{\alpha_G}{\left(\sigma_S^2 - \sigma_{AS}\right)\left(1 - c\gamma\right)} \left(1 + \frac{ac}{bG}\right) < 1$, so that ω^* and $\omega_{\min}$ are of the same sign but ω^* is smaller in

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need not fear obtaining zero wealth as tracking error risk increases. In contrast, when Cuoco and Kaniel (2001) assume that compensation also includes the manager writing a put option on his performance, the manager reduces his tracking error as performance declines. Here, compensation includes a penalty for magnitude than $\omega_{\min}$, then a decline in G which moves ω^* farther from zero and closer to $\omega_{\min}$ reduces the standard deviation of the mutual fund's return. Otherwise, a decline in G raises the standard deviation of the fund's return.

Therefore, our model does not necessarily imply the SDR relation (11), and empirical evidence against (11) found by Busse (2001) and GNW (2005) cannot rule out tournament behavior. Propositions I and II clarify that worsening performance may, indeed, cause a mutual fund manager to deviate more from the benchmark portfolio, but this could move the portfolio closer to one that minimizes standard deviation. Hence, the correct empirical indicator of risk-shifting should be the standard deviation of a fund portfolio's return relative to the benchmark's return (tracking error), not the portfolio's total return standard deviation.[18] Indeed, perhaps the most publicized "gamble" by a mutual fund manager was one that increased tracking error but reduced total return standard deviation. In late 1995, Jeffrey Vinik shifted the portfolio of Fidelity's Magellan Fund out of technology stocks and into allocations of 19% bonds and 10% cash. With the subsequent stock market rally, the bet turned sour and led to Robert Stansky replacing Vinik as the fund's portfolio manager.

We can characterize the model's prediction for the time series of a mutual fund's tracking error which will be a basis for our empirical tests. Let $R_{t+1} \equiv \ln(V_{t+1}/V_t)$ be an individual mutual fund's rate of return from the beginning of month t to the start of month t+1, and let $R_{S,t+1} \equiv$

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poor performance and compares to our case with $a < 0$, since in both instances compensation may be non-positive.

[18] Koski and Pontiff's (1999) tests calculate alternative risk measures equal to the fund return's total standard deviation, its beta, and its residual (idiosyncratic) risk. None of these measures are equal to the standard deviation of a fund's return in excess of a benchmark return (tracking error), even if the benchmark is assumed to be the market portfolio from which a fund's beta is calculated. For example, if $a > 0$ so that a fund manager increases tracking error risk as performance declines, then it can be shown that the fund's beta deviates more from unity as performance declines. That is, $\partial|\beta_V - 1|/\partial G = |\beta_A - 1|\partial|\omega|/\partial G < 0$ for $a > 0$, where β_V and β_A are the betas of the fund's portfolio and the alternative asset portfolio, respectively. However, the fund's beta or residual risk need not necessarily increase or necessarily decrease following poor performance. A derivation of this result is available from the authors upon request.

$\ln(S_{t+1}/S_t)$ be the benchmark portfolio's rate of return from the beginning of month t to the start of month t+1. Since $G_t \equiv V_t/S_t$, note that R_{t+1} - $R_{S,t+1} = \ln(V_{t+1}/V_t)$ - $\ln(S_{t+1}/S_t) = \ln(G_{t+1}/G_t) \approx d\ln G_t = dD_t$.[19] The Appendix shows that tracking error, $R_{t+1} - R_{S,t+1}$, satisfies

(14) $$R_{t+1} - R_{S,t+1} = \ln\left(G_{t+1}/G_t\right) \approx \mu_G \sqrt{h_t} - \tfrac{1}{2} h_t + \sqrt{h_t}\,\eta_{t+1}$$

where the standard deviation of tracking error equals

(15) $$\sqrt{h_t} = \left(d_0 G_t^{-1} + d_1\right)$$

and $\eta_{t+1} \sim N(0,1)$, $\mu_G \equiv |\alpha_G| / \sigma_G$, $d_0 \equiv ac|\alpha_G| / \left[b\left(1 - c\gamma\right)\sigma_G\right]$, and $d_1 \equiv |\alpha_G| / \left[\left(1 - c\gamma\right)\sigma_G\right]$. The intuition in (15) is that the standard deviation of tracking error, $\sqrt{h_t}$, is inversely related to prior performance when $d_0 > 0$, which, as shown in Proposition I, occurs when $c\gamma < 1$ and $a > 0$.

IV. Empirical Methodology

This section outlines two empirical methods that we use to examine a mutual fund's performance and its choice of risk. The first is a non-parametric test that modifies the SDR tests used in prior studies. The second is a parametric test that nests our theoretical model.

A. Standard Deviation Ratio Tests

We first replicate prior studies' test of the SDR given in equation (11). As emphasized in the preceding section, this hypothesis *is not* one that is predicted by our model. However, we then test a different SDR hypothesis that is closer in spirit to our model because it is based on the standard deviation of tracking errors rather than the standard deviation of total returns:

(16) $$\frac{\sigma_{G,2L}}{\sigma_{G,1L}} > \frac{\sigma_{G,2W}}{\sigma_{G,1W}}$$

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[19] Note that since tournaments are assumed to occur each calendar year, G_t is reset to 1 at the beginning of January each year, even though we use multiple years of fund returns to estimate these processes. Thus, G_t always equals the return on a share of the fund relative to that of the benchmark portfolio since the start of the calendar year.

where $\sigma_{G,ij}$ denotes the standard deviation of the rate of return on mutual fund *j*'s portfolio in excess of its benchmark rate of return (tracking error) during the i^{th} half of the year. Mutual fund $j = L$ is a "loser" that displayed relatively poor performance in the first half of the year, while mutual fund $j = W$ is a "winner" that had relatively good performance during the first half.

B. Parametric Tests

We propose a new time series estimation method that permits a fund's risk to respond to calendar-year performance at each (monthly) observation date, rather than just once per year. Allowing frequent changes in the fund's risk is arguably more logical since our model predicts that an optimizing manager continuously adjusts the fund's risk as its relative performance changes. Another benefit of our approach is that it allows the estimation of risk-shifting behavior for each individual mutual fund, enabling us to examine whether risk-shifting appears stronger for particular types of funds.

As with the SDR tests, our parametric tests examine the hypothesis of prior studies that a fund's standard deviation of total returns should vary inversely with its relative prior performance.[20] More importantly, we also test our model's prediction that a fund's standard deviation of tracking error varies inversely with its relative prior performance. In both cases, the parametric forms that we estimate permit a mutual fund's returns to display generalized autoregressive conditional heteroskedasticity (GARCH). Prior research has shown that the

[20] A test of this hypothesis may be of independent interest. While most research assumes that changes in a fund's risk are due to managerial incentives, Ferson and Warther (1996) offer another explanation for why a fund's standard deviation could be inversely related to its prior performance. If better performing funds receive greater cash inflows, then a fund's return standard deviation will decrease until its new (riskless) cash is fully invested in equities or until the fund increases its exposure by purchasing equity derivatives. Because transactions costs are mitigated by gradual, rather than immediate, purchase of stocks, and some mutual funds are restricted from holding derivatives, the standard deviation of a fund's returns may decline temporarily following a cash inflow. Koski and Pontiff (1999) find evidence consistent with this explanation since funds that hedge with derivatives display less risk-shifting.

returns on individual stocks and stock indices reflect GARCH-like behavior, so that a time series of equity mutual fund returns are likely to exhibit this property.[21]

We estimate our model's predicted process in (14) but modify (15) to allow tracking error variance, h_t, to change stochastically due to factors in addition to prior performance. This is done by generalizing h_t to follow an exponential GARCH (EGARCH) process first introduced by Nelson (1991):

(17) $$\ln(h_t) = a_0 + a_1 \ln(h_{t-1}) + a_2 \frac{G_t^{-1}}{\sqrt{h_{t-1}}} + a_3 |\eta_t|$$

This log variance process allows for persistence by including the lagged variable $\ln(h_{t-1})$. $\ln(h_t)$ is also influenced by the prior period's absolute innovation, $|\eta_t|$. These two variables are standard in EGARCH models. What is unique in (17) and critical for examining the effect of prior performance on a mutual fund manager's choice of risk is the variable $G_t^{-1}/\sqrt{h_{t-1}}$. As with a positive value of d_0 in (15), a positive value of a_2 in (17) supports the hypothesis that a fund manager increases tracking error volatility as the mutual fund's performance declines. A key advantage of this EGARCH specification versus (15) or a standard GARCH form is that the parameter, a_2, can be of either sign without creating the possibility that h_t could become negative. The rationale for the functional form $G_t^{-1}/\sqrt{h_{t-1}}$ is that it leads to an effect of performance on tracking error volatility that locally approximates (15) where $a_2 \approx 2d_0$.[22]

The alternative hypothesis assumed by prior studies, that a fund's prior performance predicts its future standard deviation of total returns, is tested in a similar manner. It is assumed that a mutual fund's total returns, rather than its returns in excess of a benchmark return, satisfy

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[21] For example, see French, Schwert, and Stambaugh (1987) who fit different GARCH models to the Standard & Poor's 500 stock index.

[22] Note that our theory of $\sqrt{h_t} = \left(d_0 G_t^{-1} + d_1\right)$ implies $\partial h_t / \partial G_t = -2 d_0 G_t^{-2} \sqrt{h_t}$. Taking the derivative of (15) with respect to G_t leads to $\partial h_t / \partial G_t = -a_2 G_t^{-2} h_t / \sqrt{h_{t-1}} \approx -a_2 G_t^{-2} \sqrt{h_t}$. Therefore, the parameter $a_2 \approx 2d_0$ and will be positive when $c\gamma < 1$ and $a > 0$.

$$R_{t+1} = \mu\sqrt{h_t} - \tfrac{1}{2}h_t + \sqrt{h_t}\eta_{t+1} \qquad (18)$$

where h_t is assumed to follow an EGARCH process as in (17). However, here we see that h_t is the mutual fund's total return variance, not its tracking error variance. A positive value of a_2 from joint estimation of (18) and (17) would indicate that a mutual fund manager increases the standard deviation of the fund's total return when its performance declines.

V. Data Description

Information on monthly mutual fund returns and fund characteristics comes from the Center for Research in Security Prices (CRSP) Survivor-Bias Free Mutual Fund database. The sample covers mutual funds that operated during the period from January 1962 to March 2006. We selected domestic equity funds whose investment style could be broadly classified as either a "growth" or "growth and income" fund.[23] To have sufficient observations for estimating the parameters of each mutual fund's time series processes (14), (17), and (18), we required that a mutual fund report at least 36 consecutive monthly returns. The final sample consists of 4,188 growth (G) funds, 861 growth and income (GI) funds, and 1,129 "style-mixed" (SM) funds, this last category being funds whose reported investment style was either growth or growth and income during only part of their life. Each of these three fund groups was given a different benchmark return, $R_{S,t+1} \equiv \ln(S_{t+1}/S_t)$, equal to the equally-weighted average return on all funds within the group that operated during month t.[24] By benchmarking a fund relative to others in its

[23] Mutual funds with a Wiesenberger or Investment Company Data, Inc. (ICDI) objective of "Aggressive Growth," "Growth," "Maximum Capital Gains," "Small Capitalization Growth," or "Long Term Growth," are classified as growth funds. Mutual funds with an objective of "Growth and Income" or "Growth with Current Income" are classified as growth and income funds. Index funds are excluded.

[24] This assumption implies that fund managers within a group can identify their benchmark as the "average" of the security holdings of the funds in their same group. Given that there are a large number of mutual funds in each group, this average of security holdings is likely to be close to the holdings of a style-class index. For example, managers of G funds may identify their benchmark as approximately the Russell 3000 Growth index while managers of GI funds may identify their benchmark as approximately the S&P 500 index.

broad style classification, we avoid attributing style-related differences to performance differences as would occur if the same benchmark was used for all funds.[25]

Figure 1 shows the sample's number of G, GI, and SM mutual funds in operation during each month of our sample period. G funds operating during the past decade account for a majority of our sample observations. The number of funds grew rapidly over the 1990's and peaks approximately three years prior to the sample period's end since no new funds were added during the last 36 months. This reflects the parameter estimation constraint that a fund report at least three years of returns. The proportions of all G, GI, and SM sample funds that survived (were in operation) as of March 2006 were 67%, 65%, and 62%, respectively.

Table 1 presents summary statistics of our mutual fund sample, broken down by fund style. Age is defined as the number of years from the fund's inception date until the date it expired or, for surviving funds, the end-of-sample date of March 2006. Row 1 shows that G and GI funds have, on average, similar ages of approximately 9.9 years, whereas SM funds, whose definition is conditioned on a style change having occurred, are much older. On average, G funds have greater front loads, expense ratios, and turnover ratios than GI funds. Manager tenure is the average number of years that the same individual or group manages a fund's portfolio. Average manager tenure is similar for G and GI funds, but somewhat greater for SM funds.

Table 1 also reports statistics on funds' asset size scores that measure their relative sizes by assigning a rank score from 0 (smallest) to 1 (largest) for each fund according to its total net asset value at the end of each year. This score is then averaged over each year of the fund's life. The average SM fund is larger than typical GI and G funds, likely a reflection of the greater average age of SM funds. However, on average SM funds displayed slower but less volatile asset growth, while GI funds grew slightly faster than G funds.

[25] However, in our non-parametric tests, we do examine the case of funds' performances relative to the universe of all firms, so that the benchmark is effectively the same for all funds.

The final information in Table 1 relates to a fund's number of share classes and the number of funds in its fund family. These numbers are computed each year and then averaged over all years of a fund's life. SM funds tend to have fewer share classes and belong to smaller fund families compared to G and GI funds.

VI. Results

A. Standard Deviation Ratio Test Results

Following prior studies, SDR tests are performed using an annual 2 × 2 classification of whether a fund's return performance over the first six months (RTN) was above (winner) or below (loser) the median and whether its SDR computed over the second versus first halves of the year was above or below the median, where these medians are for all funds in its style class (G, GI, or SM). First, we replicate the tests performed by past research by calculating SDR as in (11), equal to the ratio of second to first half-year standard deviations of a fund's total returns. A finding that the frequency of funds in the category (Low RTN, High SDR) significantly exceeds 25 % would be evidence in favor of the hypothesis that underperforming funds increase their total return standard deviation. Second, we calculate SDR as in (16), equal to the ratio of second to first half-year standard deviations of a fund's returns in excess of benchmark returns (tracking error).

The statistical significance of our SDR tests is determined by the method of Busse: (Busse (2001), p.58, 62-63). His method controls for the auto- and cross-correlation of fund returns that he and GNW (2005) document.[26] It involves simulating fund returns from a Fama-French-Carhart four factor model for each year and style class to obtain an empirical distribution for the 2 × 2 classifications. Specifically, for each of the 44 years in our sample, we take the N_y funds of a particular style class that operated in year y and regress each of their 12 monthly returns on market, size, book-to-market, and momentum factors. The four factors and regression

[26] The chi-squared test of significance used by BHS (1996) is valid only under the assumption that mutual fund returns are serially and cross-sectionally independent.

residuals are arranged into two matrices: a 12×4 matrix of the four factors for each month; and a $12 \times N_y$ matrix of the monthly residuals for each fund. To simulate factors, we randomly select a row from the factor matrix and use the following 11 rows in order, continuing with row one of the factor matrix after row 12. To simulate residuals, we re-sample randomly with replacement 12 rows from the residual matrix. We then create simulated monthly returns for N_y artificial funds using the betas and intercepts from the N_y regressions with the simulated factors and simulated residuals.[27]

We compute RTN and SDR (based either on total returns or tracking error) for each artificial fund and allot funds to cells in 2 × 2 contingency tables based on the median fund RTN and the median fund SDR. This procedure is repeated for each year during a particular test sample period (e.g., the entire 1962-2005 sample, or the 1995-2005 subsample) to obtain a single simulation. The entire procedure is then repeated 10,000 times to generate an empirical distribution of monthly 2 × 2 contingency table allotments under the null hypothesis of no tournament behavior. The 10 %, 5%, and 1% tails of this empirical distribution provide the significance levels for our SDR tests. Entries in Table 2 are marked by asterisks [*], [**], and [***] when results exceed these respective significance levels in the predicted direction (Low RTN, High SDR significantly greater than 25%) and by daggers [†], [††], [†††] when results are significantly opposite (Low RTN, High SDR significantly less than 25 %).

Panel A of Table 2 reports SDR test results when SDR is based on the standard deviation of total returns as in (11). Tests are performed by style class for the entire sample period, 1962-2005, and for four 11-year sub-sample periods. In addition to these separate tests by fund style, we performed tests by aggregating funds across the style classes. This aggregation was done in two ways. The first "Style Ranked" (SR) method aggregates funds based their separate style

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[27] This method preserves cross-correlation in the factor returns and the fund return residuals, as well as most of the autocorrelation in the factors. However, by using constant factor loadings throughout the year and by re-sampling the factor returns and fund return residuals, it removes any relationship between a fund's prior performance and risk.

class categorizations. For example, if a fund was categorized as (Low RTN, high SDR) based on its particular style class ranking, it remained a (Low RTN, high SDR) observation when funds were aggregated across styles to compute aggregate 2 × 2 frequencies. The second "Universe Ranked" (UR) method aggregates funds of all styles prior to calculating RTN and SDR rankings. Hence, this method essentially assumes a tournament where each Growth, Growth and Income, and Style-Mixed fund competes against all others irrespective of stated style. This aggregation method is consistent with the tests performed in Busse (2001) and GNW (2005) which assume that G, GI, and SM funds are a single style.

It is clear from Table 2 that there is no evidence of underperforming funds raising the standard deviation of their total returns in the second half of the year. The only statistically significant results occur for the 1995-2005 period and for the entire 1962-2005 sample period. But these results are exactly opposite to the findings of BHS (1996): funds that underperformed in the first half of the year tended to reduce the standard deviations of their fund returns in the second half.[28] Our results confirm those of Busse (2001) and GNW (2005) who also find evidence contrary to BHS (1996).

Panel B of Table 2 reports SDR test results when the SDR is based on the standard deviation of tracking errors as in (16). Recall that according to our theory, this type of SDR is the appropriate statistic for testing tournament behavior. Indeed we now see that there is substantial evidence that underperforming funds raise the standard deviations of their tracking errors. This behavior is statistically significant over the entire 1962-2005 sample period for G funds and for the aggregation of funds of all styles using either SR or UR rankings. Evidence of our theory's tournament behavior appears for at least some fund styles during each of the four subsamples.

[28] Note that BHS's (1996) SDR tests used Morningstar data on the returns of G mutual funds from 1980 to 1991. When we perform SDR tests using our CRSP data on the returns of G funds over the exact same period, 1980 to 1991, we match their findings. Namely, underperforming G funds raise their standard deviations of returns, and this result is statistically significant at the 5 % level even when adjusted for correlation in fund returns. As shown in Table 2 Panel A, the fact that this significance disappears when

There is no contrary evidence of the sort found in Panel A. Measuring risk appropriately as tracking error volatility, rather than total return volatility, appears to make a big difference for tests of tournament behavior.

While the results are not tabulated, we followed BHS (1996) by performing SDR tests with samples split by fund age and by fund size.[29] Similar to Table 2, Panel A underperforming funds, both new and old, in addition to small and large, did not raise the standard deviations of their total returns. The only statistically significant results were always in the opposite direction of finding that losing funds decrease their total return standard deviations. However, like Panel B of Table 2, there was evidence that underperforming funds, both new and old, in addition to small and large, raised the standard deviations of their tracking errors as our theory predicts. Evidence for this behavior was strongest for older and larger funds.

In summary, these SDR tests provide no evidence for the traditional hypothesis that underperformance leads to an increase in the standard deviation of fund returns. In contrast, there is substantially more evidence from SDR tests of an inverse relationship between performance and the standard deviation of tracking errors.

B. Parametric Test Results

The previous section's SDR tests are crude in the sense that they allow a fund's risk to change only once per year and that they require a cross-sectional grouping of funds that implicitly assumes these funds engage in similar behavior. We now perform parametric tests for each individual mutual fund that exploits the time-series properties of its returns. These tests permit a fund's risk to change at each observation date and allow risk-taking behavior to differ across funds. Maximum likelihood estimation of the EGARCH equation (17) along with either the total returns equation (18) or the tracking error equation (14) was carried out for the 4,188 G funds,

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the different periods of 1973-1983 or 1984-1994 are used underscores the fragility of the BHS (1996) findings.

[29] New (old) funds were classified as having been in existence for less (greater) than seven years. Small (large) funds were classified as below (above) the median in asset size.

861 GI funds, and 1,129 SM funds that had at least 36 monthly observations over the period from January 1962 to March 2006.[30] [31]

Table 3 reports summary statistics of the estimates of funds' total return processes, equations (18) and (17). Recall that this process is not implied by our theoretical model because, here, $\sqrt{h_t}$, represents the standard deviation of a fund's total returns and is permitted to vary with performance, G_t. The first five columns of the table give the minimum, first quartile, median, third quartile, and maximum for each parameter's point estimates for the sample of individual funds. Column six reports the proportion of estimates that is strictly greater than zero. Columns seven and eight give the proportions of estimates that are significantly positive and negative, respectively, at the five percent confidence level.

A positive value for the parameter a_2 would support the hypothesis that a fund increases its standard deviation of returns as its performance declines. However, the second-to-last row of Table 3 shows that only 34.3% of all mutual funds have a positive estimate for a_2 and the median estimate, -0.0350, is negative. 14.2 % of all funds have a significantly positive estimate of a_2, but 22.9 % have a significantly negative one. Hence, more funds appear to lower, rather than raise, the standard deviations of their returns as their performance declines. GI funds are the only style class that have marginally more estimates of a_2 that are significantly positive than are significantly negative, 18.9% versus 18.5%. However, the general results of this estimation

[30] We obtained convergence of the likelihood function for greater than 99.8% (99.9%) of the funds when estimating the total returns (tracking error) processes. For the few cases where we could not obtain convergence, the processes were estimated with the GARCH mean reversion coefficient a_1 constrained to equal the median estimate obtained from the other funds for which it was possible to find unconstrained estimates. These median values are those reported in Tables 3 and 4.

[31] As a robustness check, we also estimated each fund's total return process (18) and tracking error process (14) under the assumption that their means were constants. Specifically, the equations $R_{t+1} = c_0 + \sqrt{h_t}\eta_{t+1}$ and $R_{t+1} - R_{S,t+1} = \ln\left(G_{t+1}/G_t\right) \approx c_G + \sqrt{h_t}\eta_{t+1}$ were estimated, where c_0 and c_G are constant intercept terms. This parametric change had very little effect on the estimates of the other parameters and did not alter the qualitative results that we report below. Results of this alternative specification are available upon request.

exercise are consistent with the previous SDR tests in finding relatively few funds that raise the volatility of their returns when their performance is poor.

Table 4 presents summary statistics of the estimates of funds' tracking error processes, equations (14) and (17). In this case, $\sqrt{h_t}$ equals the standard deviation of a fund's return in excess of its style benchmark return (tracking error). A fund having a positive a_2 increases its standard deviation of tracking error as its performance declines, which is the type of tournament behavior consistent with our theory. As shown in the second-to-last row of the table, there are relatively more funds with a significantly positive value of a_2 than a significantly negative one: 16.8% versus 12.6%. The result that relatively more funds significantly raise tracking error volatility with poor performance holds for each style category. But clearly this tournament behavior is not widespread. Indeed, the majority of funds have coefficient estimates of a_2 that are insignificantly different from zero and the median point estimate is just slightly below zero. Still, the effect of tournament behavior could be sizeable for many funds. At the third quartile point estimate of $a_2 = 0.03$, a fund that was underperforming by one standard deviation ($G_t = 0.70$) would have an annual standard deviation of tracking error that was 64 basis points greater than if its performance equaled the benchmark ($G_t = 1$).[32] However, a similarly underperforming fund having the first quartile point estimate of $a_2 = -0.048$ would lower tracking error volatility by 102 basis points. Thus, underperformance can lead to economically significant rises or falls in tracking error volatility depending on the fund.

To investigate whether funds that significantly raised tracking error with underperformance ($a_2^* > 0$) differ from those that significantly lowered tracking error with underperformance ($a_2^* < 0$), we compared the characteristics of these two groups of funds. Table 5 reports the results of a univariate comparison (Panel A) and a multivariate regression analysis

(Panel B). In Panel A, column 1 gives the total number of funds for which a particular characteristic is reported, and columns 2 and 3 shows the numbers of these funds for which we obtained estimates of a_2 that were significantly positive and negative, respectively. Then, columns 4 and 5 report the average values of fund characteristics for these two groups of funds whose estimates of a_2 were significantly positive and significantly negative, respectively. Column 6 calculates the difference in the two groups' averages while column 7 reports the *t*-statistic for the test of whether the group means are statistically different.

Four of the eleven characteristics are significantly different between the two groups. Relative to a fund displaying a positive relationship between performance and tracking error, a fund displaying an inverse relationship (tournament behavior) tends to be older, larger, have fewer share classes, and have a portfolio manager with a longer tenure at the fund. That older and larger funds appear more prone to tournament behavior is consistent with our (untabulated) SDR test findings. However, because many fund characteristics are correlated and may proxy for one another, more insight regarding the determinants of tournament behavior can be obtained through multivariate regression.[33]

Panel B of Table 5 reports the results of regressions where the dependent variable is a fund's estimate of a_2 and the explanatory variables are the fund's characteristics. However, before performing these regressions, the dependent variable a_2 was winsorized at the first and 99th percentiles to eliminate the effects of extreme outliers in the estimates of this variable.[34] Columns 1 and 2 report regression coefficients and their *t*-statistics based on the sample of all 5,829 funds that reported each of the 11 characteristics. These results indicate that only a fund's age, its

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[32] The monthly standard deviation of G_t across all funds and years is 0.087 or 0.30 on an annual basis. Since from (17) $\partial\sqrt{h_t} / \partial G_t = -\frac{1}{2} a_2 / G^2$, the total change in standard deviation at G_t = 1-0.30 = 0.70 versus G_t = 1 equals $-\int_1^{0.70} \frac{1}{2} a_2 G^{-2} dt = \frac{1}{2} a_2 \left(\frac{1}{0.70} - 1\right) = 0.214 a_2$ = 0.0064.

[33] For example, the correlation between fund age and manager tenure is 0.41 across all funds and 0.42 across funds whose estimates of a_2 are statistically significant.

number of share classes, and its manager's tenure are significantly related to its estimate of a_2. However, recall that a majority of funds in this regression's sample have estimates of a_2 that are, themselves, statistically insignificant. Thus, in columns 3 and 4 of Table 5 Panel B we perform the same regression but use the sub-sample of 1,688 firms whose estimates of a_2 are statistically significant. Arguably, this sample includes only those funds that appear to actually shift their tracking error risk as their performance changes.

What we find with this sharper sample is that only a fund manager's tenure is a statistically significant determinant of the fund's tendency to increase tracking error with underperformance.[35] This result is consistent with our model's predictions. Assuming that a manager with longer tenure is more likely to accumulate personal wealth from past compensation, her fixed component of total compensation (wealth) is more likely to be positive; that is, the parameter *a* in equation (7) is positive. Furthermore, Chevalier and Ellison (1999) find that managers with longer tenures tend to have more job security, which also increases the likelihood that the manager's wealth remains positive even if the current year's performance is poor. Hence, Proposition I predicts that a manager with this characteristic (positive *a*) would tend to raise her fund's tracking error as performance declined.

In contrast, a short-tenured manager is more likely to have less personal savings and to suffer termination with poor performance. He would act as if total compensation was characterized by a negative value for parameter *a*. Our model predicts that this manager would decrease tracking error as performance declines, precisely what our empirical results confirm.

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[34] The existence of extreme outliers is clear from the minimum and maximum values of a_2 estimates in Table 4. This variable was winsorized by ranking the estimates of a_2 for all funds and then setting those estimates below (*above*) the first (*99th*) percentile to that of the first (*99th*) percentile estimate.

[35] There is a negative relationship between a fund's number of share classes and its tendency for tournament behavior, but it is statistically significant only at a 10% level. A potential explanation for this relationship is that more share classes increase the fixed administrative expense (overhead) of managing a fund, thereby making the fixed component, *a*, of the compensation function (7) tend to be negative.

VII. Conclusion

To help guide empirical tests of mutual fund tournaments, we have modeled the optimal portfolio choice of a fund manager whose compensation depends on the fund's relative performance. Perhaps the most important implication of our model is that a mutual fund's tracking error volatility, not necessarily its total return volatility, is what a portfolio manager changes as the fund's performance declines. Hence, prior studies that have found no link between a fund's performance and the standard deviation of its returns cannot be taken as conclusive evidence against tournament behavior.

Based on our model's insights, this study also developed new non-parametric and parametric methods for estimating the relation between a mutual fund's prior performance and its tracking error risk. In particular, our parametric estimation method permits a fund's risk to react to its relative performance at each observation date, not just once per calendar year. It also is able to estimate risk-shifting behavior for individual mutual funds, thereby allowing a fund's behavior to vary with its particular characteristics. We illustrated this technique by applying it to the monthly returns of more than 6,000 growth and growth and income mutual funds over the period 1962 to 2006.

As our theory predicts could happen, the paper's empirical results show that most funds do not increase their standard deviations of returns as their relative performance declines. Rather, the evidence is more supportive of the theory's prediction that underperforming funds raise their standard deviations of tracking errors. Also consistent with our model is the empirical finding that such tournament behavior is more common when fund managers have longer tenures.

Appendix

This appendix shows that equation (2) in the text is the equilibrium process for a portfolio of individual alternative securities that are chosen optimally by the fund manager. The appendix then derives the equilibrium process for a mutual fund's relative performance when the manager's compensation takes the general form given in equation (7) in the text.

As before, assume that the benchmark index follows equation (1) in the text, but let there be n different alternative securities. The date t value of the i^{th} security, $A_i(t)$, is assumed to follow the process

(A.1) $$dA_i / A_i = \alpha_i dt + \sigma_i dq_i \quad i = 1, ..., n$$

where $\sigma_S dz \sigma_i dq_i = \sigma_{Si} dt$ and $\sigma_i dq_i \; \sigma_j dq_j = \sigma_{ij} dt$ for all $i, j = 1,\ldots,n$. It is assumed that σ_S, σ_i, σ_{Si}, and σ_{ij} are constants. α_S and α_i may be time varying but the spread between their expected rates of return, $\alpha_i - \alpha_S$, is assumed to be constant.

If the fund manager allocates a portfolio proportion of w_i to alternative security i and $1-\sum_{i=1}^{n} w_i$ to the benchmark index, then the fund's portfolio value, V, follows the process

(A.2) $$\begin{aligned} dV/V &= \left(1-\sum_{i=1}^{n} w_i\right) dS/S + \sum_{i=1}^{n} w_i dA_i / A_i \\ &= \left[\left(1-\sum_{i=1}^{n} w_i\right)\alpha_S + \sum_{i=1}^{n} w_i \alpha_i\right] dt + \left[\left(1-\sum_{i=1}^{n} w_i\right)\sigma_S dz + \sum_{i=1}^{n} w_i \sigma_i dq_i\right] \end{aligned}$$

Let $w \equiv [w_1 \;\; w_2 \ldots w_n]'$ denote the $n\times 1$ vector of portfolio weights. A simple application of Itô's lemma shows that the fund's relative performance, $G \equiv V/S$, follows the process

(A.3) $$\begin{aligned} dG/G &= \sum_{i=1}^{n} w_i \left(\alpha_i - \alpha_S + \sigma_S^2 - \sigma_{Si}\right) dt + \sum_{i=1}^{n} w_i \left(\sigma_i dq_i - \sigma_S dz\right) \\ &= w'\alpha_g dt + w'\left(\sigma_g dZ\right) \end{aligned}$$

where α_g is an $n\times 1$ vector whose i^{th} element equals $\alpha_{gi} \equiv \alpha_i - \alpha_S + \sigma_S^2 - \sigma_{Si}$ and $(\sigma_g dZ)$ is an $n\times 1$ vector whose i^{th} element equals $\sigma_{gi} dz_i$, where $\sigma_{gi}^2 \equiv \sigma_i^2 + \sigma_S^2 - 2\sigma_{Si}$ and $dz_i \equiv (\sigma_i dq_i - \sigma_S dz)/\sigma_{gi}$. The Bellman equation for the manager's optimal portfolio choice problem is then

(A.4) $$0 = \underset{w}{\text{Max}}\ J_t + J_G w'\alpha_g G + \tfrac{1}{2} J_{GG} w'\Omega w G^2$$

where Ω is an $n\times n$ matrix whose i,j^{th} element equals $\Omega_{ij} \equiv \sigma_{ij} + \sigma_S^2 - \sigma_{Si} - \sigma_{Sj}$. The first order conditions with respect to the w_i 's are

(A.5) $$J_G\alpha_{gi} + J_{GG} G \sum_{j=1}^{n} w_j \Omega_{ij} = 0 \qquad i = 1,\ldots, n$$

The n linear equations in (A.5) can be solved to obtain[36]

(A.6) $$w_i = -\frac{J_G}{J_{GG} G} \sum_{j=1}^{n} v_{ij}\alpha_{gj}$$

where v_{ij} is the i,j^{th} element of Ω^{-1}. If we define δ_i as the portfolio proportion invested in alternative asset i relative to the portfolio proportion invested in all n alternative securities then

(A.7) $$\delta_i \equiv \frac{w_i}{\sum_{k=1}^{n} w_k} = \frac{\sum_{j=1}^{n} v_{ij}\alpha_{gj}}{\sum_{k=1}^{n} \sum_{j=1}^{n} v_{kj}\alpha_{gj}}$$

which is independent of the fund's relative performance, G. Since the optimal amounts invested in the individual alternative securities are constant shares of the total amount invested in alternative securities, the fund manager's problem can be simplified to one of choosing between two different assets at each point in time. One asset is the benchmark portfolio following the process in equation (1) of the text and the other is the alternative security portfolio following the

[36] Equation (A.6) holds even when one of the alternative securities is assumed to earn a risk-free return of r. For example, if the i^{th} security is risk-free, then $\alpha_{gi} \equiv r - \alpha_S + \sigma_S^2$ and $\Omega_{ij} = \sigma_S^2$. If there are no redundant alternative securities, then Ω is a positive-definite, non-singular matrix, and its inverse exists.

process given in equation (2), where $\alpha_A \equiv \sum_{i=1}^{n} \delta_i \alpha_i$, $\sigma_A^2 \equiv \sum_{i=1}^{n} \sum_{j=1}^{n} \delta_i \delta_j \sigma_{ij}$, and

$dq \equiv \sum_{i=1}^{n} \left(\delta_i \sigma_i / \sigma_A \right) dq_i$.

We next derive the equilibrium process for a mutual fund's relative performance. For a given asset allocation, the fund's relative performance follows the process of equation (4) of the text. This can be re-written as

(A.8) $$dG = \omega^* \alpha_G G dt + \omega^* G \sigma_G dx$$

where $dx = \left(\sigma_A dq - \sigma_S dz \right) / \sigma_G$ is a standard Brownian motion process and ω^* is the portfolio manager's optimal proportion invested in the alternative asset. Using equation (9) to substitute for ω^* gives

(A.9) $$dG = \left(\frac{ac}{b} + G \right) \frac{\alpha_G^2}{\left(1 - c\gamma\right) \sigma_G^2} dt + \left(\frac{ac}{b} + G \right) \frac{\alpha_G}{\left(1 - c\gamma\right) \sigma_G} dx$$

which can be re-written as

(A.10) $$dG = \mu_G \left(d_0 + d_1 G \right) dt + \left(d_0 + d_1 G \right) dx$$

where $\mu_G = \left| \alpha_G \right| / \sigma_G$, $d_0 = \dfrac{ac}{b} \dfrac{\left| \alpha_G \right|}{\left(1 - c\gamma\right) \sigma_G}$ and $d_1 = \dfrac{\left| \alpha_G \right|}{\left(1 - c\gamma\right) \sigma_G}$. With no loss in generality (dx can be redefined as $-dx$) the standard deviation in (A.9) can be assumed positive implying that $d_0 + d_1 G > 0$ and also $\mu_G = \left| \alpha_G \right| / \sigma_G > 0$.[37] Thus, one can conclude that $\operatorname{sgn}\left(d_0 \right) = \operatorname{sgn}\left(a \right)$ and $d_1 \geq 0$. Since $a > 0$ implies tracking error increases with underperformance, a positive value of d_0 implies a negative tracking error - performance relation.

Tests of (A.10) are equivalent to tests of the fund's relative (excess) return process, $d\ln G$. Using Itô's lemma and (A.10) implies

[37] Recall that $\left(\frac{ac}{b} + G \right)$ is always positive in equilibrium and an interior solution requires $c\gamma < 1$.

(A.11) $$d\ln G = \left(\frac{d_0}{G} + d_1\right)\left[\mu_G - \tfrac{1}{2}\left(\frac{d_0}{G} + d_1\right)\right]dt + \left(\frac{d_0}{G} + d_1\right)dx$$

Note that the volatility of the fund's relative return equals $\left({d_0}/{G} + d_1\right)$. Hence, under proportional compensation, $d_0 = 0$ and the volatility of the fund's relative return is independent of performance. If compensation were exponential, $d_0 > 0$ and $d_1 = 0$ so that the fund's relative return volatility would be inversely proportional to performance, G.[38]

The process in (A.11) is the continuous-time analogue of equation (14) of the text since $d\ln G = d\ln V - d\ln S \approx \ln\left(V_{t+1}/V_t\right) - \ln\left(S_{t+1}/S_t\right) \equiv R_{t+1} - R_{S,t+1}$, where R_t and R_{St} are defined as the returns on the fund's portfolio and on the benchmark portfolio, respectively. If we define the relative return volatility as $\sqrt{h_t} \equiv \left({d_0}/{G} + d_1\right)$, then in discrete time (A.11) can be approximated as

(A.12) $$R_{t+1} - R_{S,t+1} = \ln\left(G_{t+1}/G_t\right) \approx \mu_G\sqrt{h_t} - \tfrac{1}{2}h_t + \sqrt{h_t}\,\eta_{t+1}$$

where $\eta_{t+1} \sim N(0,1)$.

[38] The limiting case of exponential compensation requires $a = 1$ and $c \to \infty$, implying $d_0 > 0$ and $d_1 = 0$.

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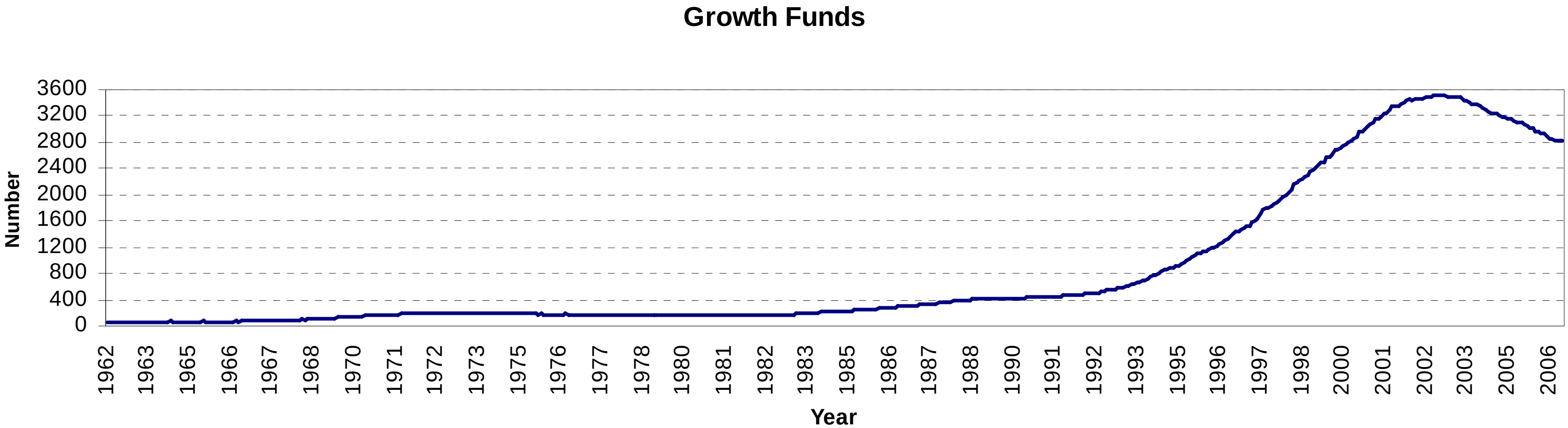
Figure 1


Growth Funds
3600
3200
2800
2400
2000
1600
1200
800
400
0
Number
1962 1963 1965 1966 1967 1968 1970 1971 1972 1973 1975 1976 1977 1978 1980 1981 1982 1983 1985 1986 1987 1988 1990 1991 1992 1993 1995 1996 1997 1998 2000 2001 2002 2003 2005 2006
Year

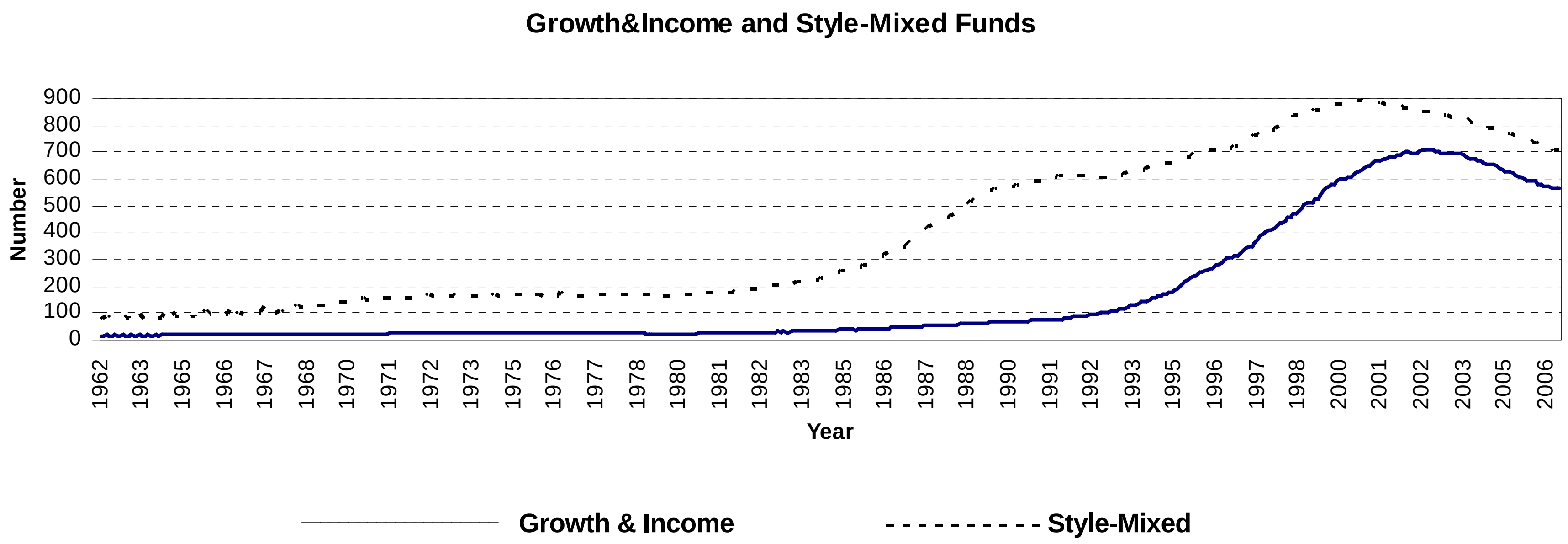

Growth&Income and Style-Mixed Funds
900
800
700
600
500
400
300
200
100
0
Number
1962 1963 1965 1966 1967 1968 1970 1971 1972 1973 1975 1976 1977 1978 1980 1981 1982 1983 1985 1986 1987 1988 1990 1991 1992 1993 1995 1996 1997 1998 2000 2001 2002 2003 2005 2006
Year
Growth & Income
Style-Mixed

Table 1 Summary Statistics of Mutual Fund Sample

Fund Characteristic	Number of Funds	Mean	Standard Deviation	Minimum	Quartiles 25%	50%	75%	Maximum
Growth Mutual Funds								
Age (years)	4188	9.92	6.89	3	6	8	11	74
Front Load	4188	0.014	0.023	0.000	0.000	0.000	0.020	0.087
Back Load	4188	0.011	0.017	0.000	0.000	0.003	0.010	0.060
Expense Ratio	4188	0.016	0.007	0.000	0.011	0.015	0.020	0.167
Turnover Ratio	4145	1.154	2.396	0.000	0.515	0.858	1.363	130.104
Manager Tenure (years)	4011	6.39	3.30	1.00	4.00	6.00	7.75	48.00
Asset Size Score	4185	0.41	0.24	0.00	0.21	0.40	0.59	0.97
Asset Growth (%)	4176	32.50	47.00	-277.80	7.39	26.25	50.98	452.18
σ of Asset Growth (%)	4138	71.88	56.16	0.22	35.58	56.09	89.69	565.11
Number of Share Classes	4188	2.78	1.39	1	1	3	4	9
Fund Family Size	4022	146.74	128.90	1	42	117	224	667
Growth and Income Mutual Funds								
Age (years)	861	9.94	9.03	3	6	8	11	82
Front Load	861	0.011	0.021	0.000	0.000	0.000	0.007	0.085
Back Load	861	0.011	0.017	0.000	0.000	0.002	0.010	0.060
Expense Ratio	860	0.014	0.006	0.000	0.009	0.013	0.019	0.035
Turnover Ratio	857	0.703	0.508	0.018	0.387	0.611	0.880	5.956
Manager Tenure (years)	847	6.31	3.10	2.00	4.33	6.00	7.50	41.00
Asset Size Score	861	0.42	0.26	0.00	0.21	0.41	0.62	0.99
Asset Growth (%)	858	34.35	45.68	-139.95	8.19	27.13	50.62	283.49
σ of Asset Growth (%)	850	69.95	58.89	0.58	31.93	53.06	85.51	513.00
Number of Share Classes	861	2.94	1.51	1	2	3	4	9
Fund Family Size	848	143.11	113.96	1	47	120	212	626
Style-Mixed Mutual Funds								
Age (years)	1129	18.29	14.50	4	9	13	21	83
Front Load	1129	0.021	0.026	0.000	0.000	0.000	0.045	0.085
Back Load	1129	0.007	0.014	0.000	0.000	0.001	0.006	0.050
Expense Ratio	1129	0.014	0.008	0.000	0.009	0.013	0.017	0.119
Turnover Ratio	1122	0.905	2.030	0.000	0.413	0.674	1.034	60.998
Manager Tenure (years)	1036	7.85	4.82	2.00	4.50	6.50	9.00	41.00
Asset Size Score	1129	0.49	0.24	0.02	0.30	0.50	0.68	0.98
Asset Growth (%)	1128	21.99	30.98	-174.47	5.42	16.98	34.10	183.41
σ of Asset Growth (%)	1126	59.39	43.06	5.32	31.82	48.97	70.52	338.83
Number of Share Classes	1129	1.93	1.24	1	1	1	3	6
Fund Family Size	1045	133.34	123.20	1	32	105	199	626

Table 1–*Continued*

Fund Characteristic	Number of Funds	Mean	Standard Deviation	Minimum	Quartiles 25%	50%	75%	Maximum
All Mutual Funds								
Age (years)	6178	11.45	9.61	3	6	9	12	83
Front Load	6178	0.015	0.023	0.000	0.000	0.000	0.030	0.087
Back Load	6178	0.010	0.017	0.000	0.000	0.002	0.010	0.060
Expense Ratio	6177	0.015	0.007	0.000	0.010	0.014	0.019	0.167
Turnover Ratio	6124	1.045	2.169	0.000	0.469	0.771	1.238	130.104
Manager Tenure (years)	5894	6.63	3.63	1.00	4.33	6.00	8.00	48.00
Asset Size Score	6175	0.43	0.24	0.00	0.22	0.42	0.62	0.99
Asset Growth (%)	6162	30.83	44.50	-277.80	7.02	23.87	47.46	452.18
σ of Asset Growth (%)	6114	69.31	54.59	0.22	34.35	54.14	85.99	565.11
Number of Share Classes	6178	2.65	1.42	1	1	3	4	9
Fund Family Size	5915	143.85	125.94	1	41	115	216	667

Note: Statistics are based on a sample taken from the CRSP Survivor-Bias Free US Mutual Fund Database covering the period January 1962 to March 2006. Mutual funds with a Wiesenberger or Investment Company Data, Inc. (ICDI) objective of "Aggressive Growth," "Growth," "Maximum Capital Gains," "Small Capitalization Growth," or "Long Term Growth," are classified as growth funds. Mutual funds with an objective of "Growth and Income" or "Growth with Current Income" are classified as growth and income funds. Style-mixed funds are mutual funds with a style of growth or growth and income for only part of their lives. Index funds as well as funds with less than 36 consecutive monthly returns are excluded. The sample contains 4188 growth funds, 861 growth-income funds, and 1129 style-mixed funds. Age is defined as the number of years from the fund's inception date until the date it expired or, for surviving funds, the end-of-sample date of March 2006. A fund's fees and turnover ratios are calculated as the average annual numbers. Manager tenure is the average number of years that an individual manages a fund's portfolio. A fund's asset size score is computed by assigning a rank score from 0(smallest) to 1(largest) for each fund according to its total net asset value at the end of each year. This score is then averaged over each year of the fund's life. A fund's asset growth equals the average annual log change in total net assets, and σ of a fund's asset growth rates is the time-series standard deviation of annual log changes in total net assets. The number of share classes records for each year the number of funds having the same fund name but with different share classes. Fund family size equals the number of mutual funds that have an identical management company name in each year. Both fund family size and numbers of fund share classes are then averaged over each year of a fund's life.

Table 2 SDR Tests of Risk-Taking Behavior

Cell frequencies are reported for a 2 x 2 classification based on a fund's: i) Standard Deviation Ratio (SDR); and (ii) Return performance for the first 6 months of each year (RTN). Funds are divided annually into four groups based on whether (i) RTN is below (low or "loser") or above (high or "winner") the median, and (ii) SDR is above (high) or below (low) the median. In Panel A, risk is defined by the total return standard deviation. In Panel B, risk is defined by the standard deviation of tracking error, using as a benchmark the equally-weighted return of funds with the same style. Results aggregated across styles are reported in two ways: "All SR" aggregates funds using the previous style-ranked categorizations of RTN and SDR; and "All UR" aggregates funds using a universe-ranked categorization of RTN and SDR. The asterisks *, **, and *** indicate 10, 5, and 1 % two-tailed p-values when the frequency of Losers with High SDR is significant in the predicted direction. The daggers †,††,††† indicate similar significance levels for the non-predicted direction. Significance is calculated for both a chi-square test (symbols following χ^2) and Busse's (2001, p.62-63) cross- and auto-correlation adjusted test (symbols following Low RTN, High SDR).

Panel A. SDR defined by the standard deviation of total returns

		Sample Frequency (% of Observations)			
		Low RTN ("Losers")		High RTN ("Winners")	
Type of Funds	Observations	Low SDR	High SDR	Low SDR	High SDR
	Sample period: 1962-1972				
Growth	1062	22.88	26.93	26.93	23.26
Growth-Income	221	20.36	28.51	28.51	22.62
Style-mixed	1226	22.68	27.00	27.00	23.33
All SR	2509	22.56	27.10	27.10	23.24
All UR	2509	22.76	27.10	27.10	23.04
	Sample period: 1973-1983				
Growth	1800	26.11	23.72	23.72	26.44
Growth-Income	282	27.30	21.63	21.63	29.43
Style-mixed	1893	26.04	23.77	23.77	26.41
All SR	3975	26.16	23.60	23.60	26.64
All UR	3975	27.22	22.74	22.74	27.30
	Sample period: 1984-1994				
Growth	4659	24.10	25.80	25.80	24.30
Growth-Income	812	23.89	25.74	25.74	24.63
Style-mixed	5317	25.18	24.77	24.77	25.28
All SR	10788	24.62	25.29	25.29	24.81
All UR	10788	24.34	25.63	25.63	24.40
	Sample period: 1995-2005				
Growth	27341	27.93	22.06†††	22.06	27.95
Growth-Income	5633	27.43	22.51†	22.51	27.55
Style-mixed	8556	27.57	22.39††	22.39	27.64
All SR	41530	27.78	22.19†††	22.19	27.83
All UR	41530	28.33	21.66†††	21.66	28.35
	Sample period: 1962-2005				
Growth	34862	27.17	22.80†††	22.80	27.24
Growth-Income	6948	26.78	23.04	23.04	27.13
Style-mixed	16992	26.30	23.62†	23.62	26.45
All SR	58802	26.87	23.06†††	23.06	27.00
All UR	58802	27.29	22.69†††	22.69	27.32

Table 2–*Continued*

Panel B. SDR defined by the standard deviation of returns in excess of benchmark (tracking error)

		Sample Frequency (% of Observations)			
		Low RTN ("Losers")		High RTN ("Winners")	
Type of Funds	Observations	Low SDR	High SDR	Low SDR	High SDR
		Sample period: 1962-1972			
Growth	1062	24.20	25.61	25.61	24.58
Growth-Income	221	20.81	28.05**	28.05	23.08
Style-mixed	1226	24.23	25.45	25.45	24.88
All SR	2509	23.91	25.75	25.75	24.59
All UR	2509	24.11	25.75	25.75	24.39
		Sample period: 1973-1983			
Growth	1800	23.72	26.11**	26.11	24.06
Growth-Income	282	25.89	23.05	23.05	28.01
Style-mixed	1893	23.45	26.36*	26.36	23.82
All SR	3975	23.75	26.01***	26.01	24.23
All UR	3975	23.60	26.36***	26.36	23.67
		Sample period: 1984-1994			
Growth	4659	23.76	26.14*	26.14	23.95
Growth-Income	812	23.03	26.60	26.60	23.77
Style-mixed	5317	24.58	25.37	25.37	24.68
All SR	10788	24.11	25.80	25.80	24.30
All UR	10788	24.07	25.90	25.90	24.13
		Sample period: 1995-2005			
Growth	27341	24.15	25.84***	25.84	24.17
Growth-Income	5633	24.69	25.24	25.24	24.82
Style-mixed	8556	24.08	25.89	25.89	24.15
All SR	41530	24.21	25.77**	25.77	24.25
All UR	41530	24.22	25.77*	25.77	24.24
		Sample period: 1962-2005			
Growth	34862	24.07	25.89***	25.89	24.15
Growth-Income	6948	24.42	25.40	25.40	24.77
Style-mixed	16992	24.18	25.75	25.75	24.33
All SR	58802	24.15	25.79***	25.79	24.27
All UR	58802	24.15	25.83***	25.83	24.18

Table 3 Parameter Estimates for Mutual Funds' Return Processes

$$R_{t+1} = \mu\sqrt{h_t} - \tfrac{1}{2}h_t + \sqrt{h_t}\eta_{t+1}$$
$$\eta_{t+1} \sim N(0,1)$$
$$\ln(h_t) = a_0 + a_1 \ln(h_{t-1}) + a_2 \frac{G_t^{-1}}{\sqrt{h_{t-1}}} + a_3\left|\eta_t\right|$$

	Distribution of Parameter Estimates					Summary Statistics		
		Quartiles				Proportion of		
	Min	25%	50%	75%	Max	Estimates>0	$t > 1.96$	$t < -1.96$
Growth Mutual Funds								
μ	-31.6042	0.0026	0.1313	0.2515	26.5735	0.755	0.288	0.043
a_0	-40.5662	-11.9096	-3.6764	-0.2916	66.6288	0.225	0.074	0.320
a_1	-5.0833	-1.4663	0.0349	0.9642	5.1631	0.509	0.261	0.199
a_2	-1.2514	-0.1477	-0.0545	0.0244	6.3555	0.321	0.125	0.245
a_3	-287.5825	-0.1093	0.1068	0.3474	9.1991	0.631	0.230	0.093
Growth-Income Mutual Funds								
μ	-4.6868	0.0697	0.2089	0.3221	33.2046	0.849	0.469	0.020
a_0	-344.1056	-12.5432	-2.7508	0.4794	18.9702	0.271	0.102	0.308
a_1	-379.9965	-1.6006	0.5618	1.3336	6.0008	0.570	0.350	0.189
a_2	-80.9025	-0.1239	-0.0183	0.0948	0.6012	0.383	0.189	0.185
a_3	-157.3799	-0.0149	0.1808	0.3521	4.5886	0.727	0.305	0.102
Style-Mixed Mutual Funds								
μ	-4.7164	0.0717	0.2214	0.3059	30.6832	0.817	0.563	0.043
a_0	-36.0890	-10.8326	-2.3014	-0.3614	15.7107	0.214	0.099	0.336
a_1	-5.5808	-0.8069	0.6350	1.0442	5.0644	0.620	0.425	0.154
a_2	-0.9953	-0.0737	-0.0119	0.0351	0.5067	0.392	0.173	0.202
a_3	-7.6088	0.0071	0.1763	0.3364	3.5623	0.758	0.395	0.081
All Mutual Funds								
μ	-31.6042	0.0189	0.1598	0.2740	33.2046	0.779	0.364	0.040
a_0	-344.1056	-11.5747	-3.3073	-0.2627	66.6288	0.229	0.083	0.321
a_1	-379.9965	-1.2882	0.2227	1.0086	6.0008	0.538	0.304	0.189
a_2	-80.9025	-0.1294	-0.0350	0.0319	6.3555	0.343	0.142	0.229
a_3	-287.5825	-0.0743	0.1371	0.3474	9.1991	0.668	0.271	0.092

Note: Maximum likelihood estimates are for the monthly return processes of 4188 Growth, 861 Growth-Income, and 1129 Style-mixed mutual funds having at least 36 monthly return observations during the January 1962 to March 2006 sample period. The benchmark returns are the equally-weighted returns of all funds, including funds not having at least 36 observations, within each investment style category. Likelihood function convergeance allowed us to obtain estimates of all parameters for 4188 Growth, 838 Growth-Income, and 1129 Style-mixed mutual funds. The value of a_1 for the remaining Growth-Income funds was fixed to the median estimate of the converged Growth-Income funds, and constrained maximum likelihood estimates of these remaining funds' other parameters were obtained.

Table 4 Parameter Estimates for Mutual Funds' Excess Return (Tracking Error) Processes

$$R_{t+1} - R_{S,t+1} = \ln\left(G_{t+1}/G_t\right) \approx \mu_G \sqrt{h_t} - \tfrac{1}{2} h_t + \sqrt{h_t}\eta_{t+1}$$
$$\eta_{t+1} \sim N(0,1)$$
$$\ln(h_t) = a_0 + a_1 \ln(h_{t-1}) + a_2 \frac{G_t^{-1}}{\sqrt{h_{t-1}}} + a_3 \left|\eta_t\right|$$

	Distribution of Parameter Estimates					Summary Statistics		
			Quartiles			Proportion of		
	Min	25%	50%	75%	Max	Estimates>0	$t > 1.96$	$t < -1.96$
Growth Mutual Funds								
μ_G	-11.3536	-0.0980	0.0079	0.1140	31.2935	0.521	0.095	0.080
a_0	-44.2961	-11.1542	-1.6548	0.4566	227.9097	0.296	0.082	0.234
a_1	-13.3081	-0.8859	0.7511	1.1768	7.5679	0.644	0.407	0.111
a_2	-1.5855	-0.0576	-0.0012	0.0288	0.7814	0.436	0.152	0.135
a_3	-991.2410	0.0119	0.2866	0.5372	7.8778	0.766	0.377	0.064
Growth-Income Mutual Funds								
μ_G	-10.7731	-0.1200	-0.0171	0.1032	16.0733	0.443	0.101	0.106
a_0	-53.8398	-9.4249	-2.2038	0.6581	33.8793	0.294	0.100	0.206
a_1	-5.2996	-0.5546	0.7961	1.1841	5.8628	0.685	0.419	0.095
a_2	-0.7395	-0.0377	-0.0005	0.0223	0.4279	0.453	0.163	0.123
a_3	-66.5036	0.0177	0.3287	0.5412	17.6268	0.774	0.417	0.079
Style-Mixed Mutual Funds								
μ_G	-2.1955	-0.0958	-0.0059	0.0762	3.0713	0.477	0.090	0.107
a_0	-36.6229	-10.2618	-1.5174	0.5645	30.1432	0.291	0.130	0.277
a_1	-4.1114	-0.4509	0.8488	1.2268	5.6245	0.693	0.507	0.121
a_2	-0.3530	-0.0174	0.0009	0.0362	0.3993	0.521	0.229	0.097
a_3	-5.0661	0.0460	0.2402	0.4075	3.7967	0.801	0.500	0.078
All Mutual Funds								
μ_G	-11.3536	-0.0993	0.0010	0.1059	31.2935	0.502	0.095	0.088
a_0	-53.8398	-11.1430	-1.6556	0.4995	227.9097	0.295	0.093	0.238
a_1	-13.3081	-0.7861	0.7705	1.1847	7.5679	0.659	0.427	0.111
a_2	-1.5855	-0.0475	-0.0001	0.0298	0.7814	0.454	0.168	0.126
a_3	-991.2410	0.0147	0.2856	0.5088	17.6268	0.773	0.405	0.068

Note: Maximum likelihood estimates are for the monthly return processes of 4188 Growth, 861 Growth-Income, and 1129 Style-mixed mutual funds having at least 36 monthly return observations during the January 1962 to March 2006 sample period. The benchmark returns are the equally-weighted returns of all funds, including funds not having at least 36 observations, within each investment style category. Likelihood function convergeance allowed us to obtain estimates of all parameters for 4188 Growth, 861 Growth-Income, and 1128 Style-mixed mutual funds. The value of a_1 for the remaining single Style-mixed fund was fixed to the median estimate of the converged Style-mixed funds, and constrained maximum likelihood estimates for this remaining fund's other parameters were obtained.

Table 5 Characteristics of Risk-Shifting Mutual Funds

$$R_{t+1} - R_{S,t+1} = \ln\left(G_{t+1}/G_t\right) \approx \mu_G \sqrt{h_t} - \tfrac{1}{2} h_t + \sqrt{h_t}\eta_{t+1}$$
$$\eta_{t+1} \sim N(0,1)$$
$$\ln(h_t) = a_0 + a_1 \ln(h_{t-1}) + a_2 \frac{G_t^{-1}}{\sqrt{h_{t-1}}} + a_3 \left|\eta_t\right|$$

Panel A. Univariate Analysis

Fund Characteristic	Number Of Funds	Number of Funds $a_2^* > 0$	$a_2^* < 0$	Characteristic Average $a_2^* > 0$	$a_2^* < 0$	Difference	t-value
Age (years)	6178	1036	780	12.571	10.503	2.069***	4.19
Front Load	6178	1036	780	0.016	0.015	0.001	1.11
Back Load	6178	1036	780	0.010	0.010	0.000	-0.28
Expense Ratio	6177	1036	780	0.015	0.015	0.000	-0.82
Turnover Ratio	6124	1027	770	1.192	1.096	0.096	0.56
Manager Tenure (years)	5894	981	732	6.948	6.022	0.926***	5.09
Asset Size Score	6175	1036	779	0.428	0.402	0.026**	2.27
Asset Growth (%)	6162	1034	776	31.338	32.237	-0.900	-0.41
σ of Asset Growth (%)	6114	1025	766	70.151	72.779	-2.628	-0.97
Number of Share Classes	6178	1036	780	2.545	2.723	-0.179***	-2.61
Fund Family Size	5915	988	734	144.45	150.65	-6.19	-0.97

Panel B. Multivariate Regression Analysis

Independent Variables	Dependent Variable: a_2 All Funds Coefficient	t-value	Funds with Significant a_2 Coefficient	t-value
Age (years)	4.44E-04***	2.75	1.55E-04	0.44
Front Load	2.84E-02	0.47	1.55E-01	1.12
Back Load	1.80E-02	0.20	5.88E-03	0.03
Expense Ratio	3.34E-01	1.47	7.29E-01	1.36
Turnover Ratio	2.28E-04	0.40	3.26E-04	0.40
Manager Tenure (years)	8.51E-04**	2.21	2.17E-03**	2.47
Asset Size Score	-1.09E-03	-0.18	8.49E-03	0.62
Asset Growth (%)	4.93E-06	0.15	-5.05E-05	-0.60
σ of Asset Growth (%)	-2.21E-05	-0.83	1.10E-05	0.17
Number of Share Classes	-3.63E-03***	-3.26	-4.54E-03*	-1.75
Fund Family Size	1.59E-05	1.38	1.71E-05	0.65
Adjusted R^2	0.65		0.59	
# Observations	5829		1688	

Note: For the regression analysis, the dependent variable a_2 was winsorized to remove the effects of extreme outliers. This was done by ranking the estimates of a_2 for all funds and setting those above the 99^{th} percentile to the value of a_2 at the 99^{th} percentile. Similarly, all of the values of a_2 below the first percentile were set to the value of a_2 at the first percentile.