Technical Appendix to

"Harming Depositors and Helping Borrowers:

The Disparate Impact of Bank Consolidation"

(Not to be published but to be made available upon request.)

Details of Proofs of Propositions 1 and 2

To derive Proposition 1's exact and sufficient conditions for a greater number of LMBs, k, to reduce the retail loan rates of small banks in market M, we can directly differentiate equation (26) or, alternatively, differentiate equation (22) for the case of a small bank in market M:

$$\frac{\partial r_{L,i}^{M}}{\partial k} = -\frac{\partial \delta_{i,m/k}}{\partial k}\left[r_E + c_L + t_L \tfrac{1}{m} - r_{L,1}\right] + \delta_{i,m/k}\frac{\partial r_{L,1}}{\partial k} \tag{T.1}$$

To find $\partial r_{L,1}/\partial k$, we differentiate $r_{L,1}$ in equation (25) to obtain

$$\frac{\partial r_{L,1}}{\partial k} = -\frac{\left[\beta_{m/k}\frac{\partial \beta_{n/k}}{\partial k} - \beta_{n/k}\frac{\partial \beta_{m/k}}{\partial k}\right]}{\left(\beta_{n/k}L^N + \beta_{m/k}L^M\right)^2} t_L L^N L^M \left(\frac{1}{m} - \frac{1}{n}\right) + \frac{\Lambda\left(L^N + L^M\right)\left(L^N \frac{\partial \beta_{n/k}}{\partial k} + L^M \frac{\partial \beta_{m/k}}{\partial k}\right)}{\left(\beta_{n/k}L^N + \beta_{m/k}L^M\right)^2} \tag{T.2}$$

Substituting in for $\partial r_{L,1}/\partial k$ and $r_{L,1}$ from (T.2) and equation (25), (T.1) becomes

$$\begin{aligned}\frac{\partial r_{L,i}^{M}}{\partial k} = &-\frac{\frac{\partial \delta_{i,m/k}}{\partial k}\left[\Lambda\left(L^N + L^M\right) + \beta_{n/k}L^N t_L\left(\tfrac{1}{m} - \tfrac{1}{n}\right)\right]}{\beta_{n/k}L^N + \beta_{m/k}L^M} \\ &+ \frac{\delta_{i,m/k}L^N\left\{\frac{\partial \beta_{n/k}}{\partial k}\left[\Lambda\left(L^N + L^M\right) - \beta_{m/k}t_L L^M\left(\frac{1}{m} - \frac{1}{n}\right)\right]\right\}}{\left(\beta_{n/k}L^N + \beta_{m/k}L^M\right)^2} \\ &+ \frac{\delta_{i,m/k}L^M\left\{\frac{\partial \beta_{m/k}}{\partial k}\left[\Lambda\left(L^N + L^M\right) + \beta_{n/k}t_L L^N\left(\frac{1}{m} - \frac{1}{n}\right)\right]\right\}}{\left(\beta_{n/k}L^N + \beta_{m/k}L^M\right)^2}\end{aligned} \tag{T.3}$$

Re-arranging the right-hand side of (T.3), one finds that it is negative when

$$\begin{aligned}0 > &-t_L\left(\tfrac{1}{m} - \tfrac{1}{n}\right)L^N \beta_{n/k}\left\{\frac{\partial \delta_{i,m/k}}{\partial k}\left(\beta_{n/k}L^N + \beta_{m/k}L^M\right) + \delta_{i,m/k}L^M\left(\frac{\partial \beta_{n/k}}{\partial k}\frac{\beta_{m/k}}{\beta_{n/k}} - \frac{\partial \beta_{m/k}}{\partial k}\right)\right\} \\ &-\Lambda\left(L^N + L^M\right)\left\{\frac{\partial \delta_{i,m/k}}{\partial k}\left(\beta_{n/k}L^N + \beta_{m/k}L^M\right) - \delta_{i,m/k}\left(L^N \frac{\partial \beta_{n/k}}{\partial k} + L^M \frac{\partial \beta_{m/k}}{\partial k}\right)\right\}\end{aligned} \tag{T.4}$$

or

$$\Lambda > -\frac{t_L\left(\frac{1}{m}-\frac{1}{n}\right)L^N\beta_{n/k}}{L^N+L^M}\left[\frac{\frac{\partial\delta_{i,m/k}}{\partial k}\left(\beta_{n/k}L^N+\beta_{m/k}L^M\right)+\delta_{i,m/k}L^M\left(\frac{\partial\beta_{n/k}}{\partial k}\frac{\beta_{m/k}}{\beta_{n/k}}-\frac{\partial\beta_{m/k}}{\partial k}\right)}{\frac{\partial\delta_{i,m/k}}{\partial k}\left(\beta_{n/k}L^N+\beta_{m/k}L^M\right)-\delta_{i,m/k}\left(L^N\frac{\partial\beta_{n/k}}{\partial k}+L^M\frac{\partial\beta_{m/k}}{\partial k}\right)}\right] \quad (T.5)$$

Recall that $\beta_{n/k} = (2 - \delta_{2,n/k}) > 0$ and $\partial\beta_{n/k}/\partial k = - \partial\delta_{2,n/k}/\partial k < 0$. By substituting in for $\delta_{2,n/k}$ from equation (24) in the paper, it is straightforward to show that $(\beta_{m/k}\,\partial\beta_{n/k}/\partial k - \beta_{n/k}\,\partial\beta_{m/k}/\partial k) > 0$ for $n > m$. This, along with the fact that $\partial\delta_{i,m/k}/\partial k > 0$ indicates that the numerator of the term in brackets is positive. In addition, one can see that the denominator is positive, so that the ratio in brackets is positive. Now by re-writing this ratio, (T.5) can be re-written as

$$\Lambda > -\frac{t_L\left(\frac{1}{m}-\frac{1}{n}\right)L^N\beta_{n/k}}{L^N+L^M}\left[\frac{\Psi^L_{i,m}+\delta_{i,m/k}L^M\frac{\partial\beta_{n/k}}{\partial k}\left(\frac{\beta_{m/k}}{\beta_{n/k}}+\frac{L^N}{L^M}\right)}{\Psi^L_{i,m}}\right] \quad (T.6)$$

where $\Psi^L_{i,m} \equiv \frac{\partial\delta_{i,m/k}}{\partial k}\left(\beta_{n/k}L^N+\beta_{m/k}L^M\right)-\delta_{i,m/k}\left(L^N\frac{\partial\beta_{n/k}}{\partial k}+L^M\frac{\partial\beta_{m/k}}{\partial k}\right)>0$. In (T.6), since $\partial\beta_{n/k}/\partial k < 0$, then $\delta_{i,m/k}L^M\frac{\partial\beta_{n/k}}{\partial k}\left(\frac{\beta_{m/k}}{\beta_{n/k}}+\frac{L^N}{L^M}\right)<0$, which permits us to show that $1 \geq \left[\Psi^L_{i,m}+\delta_{i,m/k}L^M\frac{\partial\beta_{n/k}}{\partial k}\left(\frac{\beta_{m/k}}{\beta_{n/k}}+\frac{L^N}{L^M}\right)\right]/\Psi^L_{i,m} > 0$. Thus, a sufficient condition for $\partial r^M_{L,i}/\partial k < 0$ is $\Lambda > 0$.

Similarly, to find the exact and sufficient conditions for a greater number of LMBs, k, to reduce the retail loan rates of small banks in market N, we can differentiate equation (22) for the case of a small bank in market N:

$$\frac{\partial r^N_{L,i}}{\partial k} = -\frac{\partial\delta_{i,n/k}}{\partial k}\left[r_E + c_L + t_L\tfrac{1}{n} - r_{L,1}\right] + \delta_{i,n/k}\frac{\partial r_{L,1}}{\partial k} \quad (T.7)$$

Substituting in for $\partial r_{L,1}/\partial k$ and $r_{L,1}$ from (T.2) and equation (25), (T.7) becomes

$$\frac{\partial r_{L,i}^{N}}{\partial k} = -\frac{\frac{\partial \delta_{i,n/k}}{\partial k}\left[\Lambda\left(L^{N}+L^{M}\right)-\beta_{m/k}L^{M}t_{L}\left(\frac{1}{m}-\frac{1}{n}\right)\right]}{\beta_{n/k}L^{N}+\beta_{m/k}L^{M}}$$

$$+\frac{\delta_{i,n/k}L^{N}\left\{\frac{\partial \beta_{n/k}}{\partial k}\left[\Lambda\left(L^{N}+L^{M}\right)-\beta_{m/k}t_{L}L^{M}\left(\frac{1}{m}-\frac{1}{n}\right)\right]\right\}}{\left(\beta_{n/k}L^{N}+\beta_{m/k}L^{M}\right)^{2}} \tag{T.8}$$

$$+\frac{\delta_{i,n/k}L^{M}\left\{\frac{\partial \beta_{m/k}}{\partial k}\left[\Lambda\left(L^{N}+L^{M}\right)+\beta_{n/k}t_{L}L^{N}\left(\frac{1}{m}-\frac{1}{n}\right)\right]\right\}}{\left(\beta_{n/k}L^{N}+\beta_{m/k}L^{M}\right)^{2}}$$

This derivative is negative when

$$0 > L^{M}t_{L}\left(\tfrac{1}{m}-\tfrac{1}{n}\right)\left\{\frac{\partial \delta_{i,n/k}}{\partial k}\left(\beta_{n/k}L^{N}+\beta_{m/k}L^{M}\right)\beta_{m/k}+\delta_{i,n/k}L^{N}\left(\frac{\partial \beta_{m/k}}{\partial k}\beta_{n/k}-\frac{\partial \beta_{n/k}}{\partial k}\beta_{m/k}\right)\right\}$$
$$-\Lambda\left(L^{N}+L^{M}\right)\left\{\frac{\partial \delta_{i,n/k}}{\partial k}\left(\beta_{n/k}L^{N}+\beta_{m/k}L^{M}\right)-\delta_{i,n/k}\left(L^{N}\frac{\partial \beta_{n/k}}{\partial k}+L^{M}\frac{\partial \beta_{m/k}}{\partial k}\right)\right\} \tag{T.9}$$

or

$$\Lambda > \frac{L^{M}t_{L}\left(\frac{1}{m}-\frac{1}{n}\right)\beta_{m/k}}{L^{N}+L^{M}}\left[\frac{\frac{\partial \delta_{i,n/k}}{\partial k}\left(\beta_{n/k}L^{N}+\beta_{m/k}L^{M}\right)+\delta_{i,n/k}L^{N}\left(\frac{\beta_{n/k}}{\beta_{m/k}}\frac{\partial \beta_{m/k}}{\partial k}-\frac{\partial \beta_{n/k}}{\partial k}\right)}{\frac{\partial \delta_{i,n/k}}{\partial k}\left(\beta_{n/k}L^{N}+\beta_{m/k}L^{M}\right)-\delta_{i,n/k}\left(L^{N}\frac{\partial \beta_{n/k}}{\partial k}+L^{M}\frac{\partial \beta_{m/k}}{\partial k}\right)}\right] \tag{T.10}$$

which can also be written as

$$\Lambda > \frac{L^{M}t_{L}\left(\frac{1}{m}-\frac{1}{n}\right)\beta_{m/k}}{L^{N}+L^{M}}\left[\frac{\Psi_{i,n}^{L}+\delta_{i,n/k}L^{N}\frac{\partial \beta_{m/k}}{\partial k}\left(\frac{\beta_{n/k}}{\beta_{m/k}}+\frac{L^{M}}{L^{N}}\right)}{\Psi_{i,n}^{L}}\right] \tag{T.11}$$

where $\Psi_{i,n}^{L} \equiv \frac{\partial \delta_{i,n/k}}{\partial k}\left(\beta_{n/k}L^{N}+\beta_{m/k}L^{M}\right)-\delta_{i,n/k}\left(L^{N}\frac{\partial \beta_{n/k}}{\partial k}+L^{M}\frac{\partial \beta_{m/k}}{\partial k}\right)>0$. Since $\partial\beta_{m/k}/\partial k < 0$, then $\delta_{i,n/k}L^{N}\frac{\partial \beta_{m/k}}{\partial k}\left(\frac{\beta_{n/k}}{\beta_{m/k}}+\frac{L^{M}}{L^{N}}\right)<0$. This implies that the term in brackets in (T.11) is less than one. Hence, a sufficient condition for $\partial r_{L,i}^{N}/\partial k < 0$ is $\Lambda > \frac{L^{M}}{L^{N}+L^{M}}\left(\frac{1}{m}-\frac{1}{n}\right)t_{L}\beta_{m/k}$.

To derive Proposition 2's exact and sufficient conditions for a greater number of LMBs, *k*, to reduce the retail deposit rates of small banks in market *M*, we can directly differentiate equation (29) or, alternatively, differentiate equation (23) for the case of a small bank in market *M*:

$$\frac{\partial r_{D,i}^{M}}{\partial k} = -\frac{\partial \delta_{i,m/k}}{\partial k}\left[r_E - c_D - t_D \tfrac{1}{m} - r_{D,1}\right] + \delta_{i,m/k}\frac{\partial r_{D,1}}{\partial k} \tag{T.12}$$

To find $\partial r_{D,1}/\partial k$, we differentiate $r_{D,1}$ in equation (28) to obtain

$$\frac{\partial r_{D,1}}{\partial k} = \frac{\left[\beta_{m/k}\frac{\partial \beta_{n/k}}{\partial k} - \beta_{n/k}\frac{\partial \beta_{m/k}}{\partial k}\right]}{\left(\beta_{n/k}D^N + \beta_{m/k}D^M\right)^2} t_D D^N D^M \left(\tfrac{1}{m} - \tfrac{1}{n}\right) + \frac{\Delta\left(D^N + D^M\right)\left(D^N\frac{\partial \beta_{n/k}}{\partial k} + D^M\frac{\partial \beta_{m/k}}{\partial k}\right)}{\left(\beta_{n/k}D^N + \beta_{m/k}D^M\right)^2} \tag{T.13}$$

Substituting in for $\partial r_{D,1}/\partial k$ and $r_{D,1}$ from (T.13) and equation (28), (T.12) becomes

$$\begin{aligned}\frac{\partial r_{D,i}^{M}}{\partial k} = &-\frac{\frac{\partial \delta_{i,m/k}}{\partial k}\left[\Delta\left(D^N + D^M\right) - \beta_{n/k}D^N t_D\left(\tfrac{1}{m} - \tfrac{1}{n}\right)\right]}{\beta_{n/k}D^N + \beta_{m/k}D^M} \\ &+ \frac{\delta_{i,m/k}D^M\left\{\frac{\partial \beta_{m/k}}{\partial k}\left[\Delta\left(D^N + D^M\right) - \beta_{n/k}t_D D^N\left(\frac{1}{m} - \frac{1}{n}\right)\right]\right\}}{\left(\beta_{n/k}D^N + \beta_{m/k}D^M\right)^2} \\ &+ \frac{\delta_{i,m/k}D^N\left\{\frac{\partial \beta_{n/k}}{\partial k}\left[\Delta\left(D^N + D^M\right) + \beta_{m/k}t_D D^M\left(\frac{1}{m} - \frac{1}{n}\right)\right]\right\}}{\left(\beta_{n/k}D^N + \beta_{m/k}D^M\right)^2}\end{aligned} \tag{T.14}$$

This derivative is negative when

$$\begin{aligned}0 > &\, t_D D^N\left(\tfrac{1}{m} - \tfrac{1}{n}\right)\left\{\frac{\partial \delta_{i,m/k}}{\partial k}\left(\beta_{n/k}D^N + \beta_{m/k}D^M\right)\beta_{n/k} + \delta_{i,m/k}D^M\left(-\frac{\partial \beta_{m/k}}{\partial k}\beta_{n/k} + \frac{\partial \beta_{n/k}}{\partial k}\beta_{m/k}\right)\right\} \\ &- \Delta\left(D^N + D^M\right)\left\{\frac{\partial \delta_{i,m/k}}{\partial k}\left(\beta_{n/k}D^N + \beta_{m/k}D^M\right) - \delta_{i,m/k}\left(D^M\frac{\partial \beta_{m/k}}{\partial k} + D^N\frac{\partial \beta_{n/k}}{\partial k}\right)\right\}\end{aligned} \tag{T.15}$$

or

$$\Delta > \frac{t_D D^N\left(\tfrac{1}{m} - \tfrac{1}{n}\right)\beta_{n/k}}{D^N + D^M}\left[\frac{\frac{\partial \delta_{i,m/k}}{\partial k}\left(\beta_{n/k}D^N + \beta_{m/k}D^M\right) + \delta_{i,m/k}D^M\left(\frac{\partial \beta_{n/k}}{\partial k}\frac{\beta_{m/k}}{\beta_{n/k}} - \frac{\partial \beta_{m/k}}{\partial k}\right)}{\frac{\partial \delta_{i,m/k}}{\partial k}\left(\beta_{n/k}D^N + \beta_{m/k}D^M\right) - \delta_{i,m/k}\left(D^M\frac{\partial \beta_{m/k}}{\partial k} + D^N\frac{\partial \beta_{n/k}}{\partial k}\right)}\right] \tag{T.16}$$

which can be written as

$$\Delta > \frac{t_D D^N\left(\tfrac{1}{m} - \tfrac{1}{n}\right)\beta_{n/k}}{D^N + D^M}\left[\frac{\Psi_{i,m}^{D} + \delta_{i,m/k}D^M\frac{\partial \beta_{n/k}}{\partial k}\left(\frac{\beta_{m/k}}{\beta_{n/k}} + \frac{D^N}{D^M}\right)}{\Psi_{i,m}^{D}}\right] \tag{T.17}$$

where $\Psi^{D}_{i,m} \equiv \frac{\partial \delta_{i,m/k}}{\partial k}\left(\beta_{n/k}D^{N}+\beta_{m/k}D^{M}\right)-\delta_{i,m/k}\left(D^{N}\frac{\partial \beta_{n/k}}{\partial k}+D^{M}\frac{\partial \beta_{m/k}}{\partial k}\right)>0$. Since $\partial\beta_{n/k}/\partial k < 0$, then $\delta_{i,m/k}D^{M}\frac{\partial \beta_{n/k}}{\partial k}\left(\frac{\beta_{m/k}}{\beta_{n/k}}+\frac{D^{N}}{D^{M}}\right)<0$. This implies the term in brackets in (T.17) is less than one. Hence, a sufficient condition for $\partial r^{M}_{D,i}/\partial k < 0$ is $\Delta > \frac{D^{N}}{D^{N}+D^{M}}\left(\frac{1}{m}-\frac{1}{n}\right)t_{D}\beta_{n/k}$.

Similarly, to find the exact and sufficient conditions for a greater number of LMBs, k, to reduce the retail deposit rates of small banks in market N, we can differentiate equation (23) for the case of a small bank in market N:

$$\frac{\partial r^{N}_{D,i}}{\partial k}=-\frac{\partial \delta_{i,n/k}}{\partial k}\left[r_{E}-c_{D}-t_{D}\tfrac{1}{n}-r_{D,1}\right]+\delta_{i,n/k}\frac{\partial r_{D,1}}{\partial k} \tag{T.18}$$

Substituting in for $\partial r_{D,1}/\partial k$ and $r_{D,1}$ from (T.13) and equation (28), (T.18) becomes

$$\begin{aligned}\frac{\partial r^{N}_{D,i}}{\partial k}=&-\frac{\frac{\partial \delta_{i,n/k}}{\partial k}\left[\Delta\left(D^{N}+D^{M}\right)+\beta_{m/k}D^{M}t_{D}\left(\frac{1}{m}-\frac{1}{n}\right)\right]}{\beta_{n/k}D^{N}+\beta_{m/k}D^{M}}\\&+\frac{\delta_{i,n/k}D^{N}\left\{\frac{\partial \beta_{n/k}}{\partial k}\left[\Delta\left(D^{N}+D^{M}\right)+\beta_{m/k}t_{D}D^{M}\left(\frac{1}{m}-\frac{1}{n}\right)\right]\right\}}{\left(\beta_{n/k}D^{N}+\beta_{m/k}D^{M}\right)^{2}}\\&+\frac{\delta_{i,n/k}D^{M}\left\{\frac{\partial \beta_{m/k}}{\partial k}\left[\Delta\left(D^{N}+D^{M}\right)-\beta_{n/k}t_{D}D^{N}\left(\frac{1}{m}-\frac{1}{n}\right)\right]\right\}}{\left(\beta_{n/k}D^{N}+\beta_{m/k}D^{M}\right)^{2}}\end{aligned} \tag{T.19}$$

This derivative is negative when

$$\begin{aligned}0>&-t_{D}\left(\tfrac{1}{m}-\tfrac{1}{n}\right)D^{M}\beta_{m/k}\left[\frac{\partial \delta_{i,n/k}}{\partial k}\left(\beta_{n/k}D^{N}+\beta_{m/k}D^{M}\right)-D^{N}\delta_{i,n/k}\left(\frac{\partial \beta_{n/k}}{\partial k}-\frac{\partial \beta_{m/k}}{\partial k}\frac{\beta_{n/k}}{\beta_{m/k}}\right)\right]\\&-\Delta\left(D^{N}+D^{M}\right)\left[\frac{\partial \delta_{i,n/k}}{\partial k}\left(\beta_{n/k}D^{N}+\beta_{m/k}D^{M}\right)-\delta_{i,n/k}\left(D^{N}\frac{\partial \beta_{n/k}}{\partial k}+D^{M}\frac{\partial \beta_{m/k}}{\partial k}\right)\right]\end{aligned} \tag{T.20}$$

or

$$\Delta>-\frac{t_{D}\left(\frac{1}{m}-\frac{1}{n}\right)D^{M}\beta_{m/k}}{D^{N}+D^{M}}\left[\frac{\frac{\partial \delta_{i,n/k}}{\partial k}\left(\beta_{n/k}D^{N}+\beta_{m/k}D^{M}\right)-\delta_{i,n/k}D^{N}\left(\frac{\partial \beta_{n/k}}{\partial k}-\frac{\partial \beta_{m/k}}{\partial k}\frac{\beta_{n/k}}{\beta_{m/k}}\right)}{\frac{\partial \delta_{i,n/k}}{\partial k}\left(\beta_{n/k}D^{N}+\beta_{m/k}D^{M}\right)-\delta_{i,n/k}\left(D^{N}\frac{\partial \beta_{n/k}}{\partial k}+D^{M}\frac{\partial \beta_{m/k}}{\partial k}\right)}\right] \tag{T.21}$$

which can also be written as

$$\Delta > -\frac{t_D\left(\frac{1}{m}-\frac{1}{n}\right)D^M\beta_{m/k}}{D^N+D^M}\left[\frac{\Psi_{i,n}^D+\delta_{i,n/k}D^N\frac{\partial\beta_{m/k}}{\partial k}\left(\frac{\beta_{n/k}}{\beta_{m/k}}+\frac{D^M}{D^N}\right)}{\Psi_{i,n}^D}\right] \tag{T.22}$$

where $\Psi_{i,n}^D \equiv \frac{\partial\delta_{i,n/k}}{\partial k}\left(\beta_{n/k}D^N+\beta_{m/k}D^M\right)-\delta_{i,n/k}\left(D^N\frac{\partial\beta_{n/k}}{\partial k}+D^M\frac{\partial\beta_{m/k}}{\partial k}\right)>0$. Since $\partial\beta_{m/k}/\partial k < 0$, then $\delta_{i,n/k}D^N\frac{\partial\beta_{m/k}}{\partial k}\left(\frac{\beta_{n/k}}{\beta_{m/k}}+\frac{D^M}{D^N}\right)<0$. This implies the term in brackets in (T.22) or (T.21) is less than one. Indeed, for some values of D^N and D^M, this term in brackets could even become negative in which case we would need $\Delta > 0$ for $\partial r_{D,i}^N/\partial k<0$. This is because, as mentioned earlier, $(\beta_{m/k}\,\partial\beta_{n/k}/\partial k - \beta_{n/k}\,\partial\beta_{m/k}/\partial k) > 0$ for $n > m$ so that the term

$$\delta_{i,n/k}D^N\left(\frac{\partial\beta_{n/k}}{\partial k}-\frac{\partial\beta_{m/k}}{\partial k}\frac{\beta_{n/k}}{\beta_{m/k}}\right)>0$$

and the numerator of the term in brackets in (T.21) can become negative for large D^N. However, from inspection of $\partial r_{D,i}^N/\partial k$ in (T.19) one can see that the first two terms are negative when $\Delta > -\frac{D^M}{D^N+D^M}\left(\frac{1}{m}-\frac{1}{n}\right)t_D\beta_{m/k}$ and the third term is negative when $\Delta > \frac{D^N}{D^N+D^M}\left(\frac{1}{m}-\frac{1}{n}\right)t_D\beta_{n/k}$. Hence, a sufficient condition for $\partial r_{D,i}^N/\partial k<0$ is $\Delta > \frac{D^N}{D^N+D^M}\left(\frac{1}{m}-\frac{1}{n}\right)t_D\beta_{n/k}$.