



20 - 1:

A. The monthly mortgage payment is \$2,306.74

$$\begin{aligned} \text{Monthly payment} &= \text{Principal} \left( \frac{r (1+r)^m}{(1+r)^m - 1} \right) \\ &= \$300,000 \left( \frac{\frac{0.085}{12} \left( 1 + \frac{0.085}{12} \right)^{360}}{\left( 1 + \frac{0.085}{12} \right)^{360} - 1} \right) = \$2,306.74 \end{aligned}$$

B. On the first payment \$2,125.00 is interest  $\left[ \$300,000 \left( \frac{0.085}{12} \right) \right]$  and the remainder, \$181.74, is principal.

C. The first payment reduces or pays down the principal by \$181.74. This means you still owe \$299,818.26 on your mortgage. The second payment will thus include interest of \$2123.71  $\left[ \$299,818.26 \left( \frac{0.085}{12} \right) \right]$  and \$183.03 principal.

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20 - 2:

If the current mortgage rates are lower than 7.3% (6.8% + 0.5%) then the dealer wants to keep the securities with the lowest probability of prepayment because these mortgages generate a higher than current rate of interest. 250% PSA means that, even when mortgage rates remain steady, this pool will prepay faster than the PSA benchmark pool (perhaps the mortgages are all in Champaign-Urbana where, due to the University, mobility is high). By slipping your colleague the highest %PSA pool he has on his books he reduces his own **prepayment risk exposure**.

When the current mortgage rates are higher than 7.3% then the dealer wants to keep the securities with the highest probability of prepayment because he can reinvest the proceeds at a higher rate. By slipping your colleague the lowest %PSA pools on his book the dealer reduces his **extension risk exposure**.

Naturally, when you place orders with the dealer, you specify the %PSA on the mortgage pool before you buy or sell anything.

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## 20 - 3:

- A. John Q. Investor purchased \$100,000 of a \$1m pool of mortgages. This means that JQ owns 1/10 of the each mortgage in the pool. If the outstanding principal remaining is \$700,000 then JQ's portion is \$70,000.
- B. The scheduled principal payment in month 31 is \$1,600 then JQ's share is 1/10th or \$160.00
- C. We translate the description on page 311 into mathematics as follows:

$$Rate = PSA \left( 0.06 \left( \min \left( 1, \frac{months}{30} \right) \right) \right)$$

For 100% PSA we calculate the expected prepayment rate in month 31 as 6%

$$\begin{aligned} Rate &= PSA \left( 0.06 \left( \min \left( 1, \frac{months}{30} \right) \right) \right) \\ &= 1.00 \left( 0.06 \left( \min \left( 1, \frac{31}{30} \right) \right) \right) \\ &= 0.06 \end{aligned}$$

- D. We calculate the expected prepayment as \$359.19

$$\begin{aligned} E[P] &= \left[ 1 \& \left( 1 \& Prepayment\ Rate \right)^{1/12} \right] \left( Outstanding\ Principal \& Scheduled\ Principal\ Payment \right) \\ &= \left[ 1 \& \left( 1 \& 0.06 \right)^{1/12} \right] \left( \$70,000.00 \& \$160.00 \right) \\ &= 0.005143013 \left( \$69,840 \right) \\ &= \$359.19 \end{aligned}$$

- E. At 80% PSA the prepayment rate is 4.8% instead of 6%

$$\begin{aligned} Rate &= PSA \left( 0.06 \left( \min \left( 1, \frac{months}{30} \right) \right) \right) \\ &= 0.80 \left( 0.06 \left( \min \left( 1, \frac{31}{30} \right) \right) \right) \\ &= 0.048 \end{aligned}$$

and the expected pre-payment is \$285.70 instead of \$359.19

$$\begin{aligned} E[P] &= \left[ 1 \& \left( 1 \& Prepayment\ Rate \right)^{1/12} \right] \left( Outstanding\ Principal \& Scheduled\ Principal\ Payment \right) \\ &= \left[ 1 \& \left( 1 \& 0.048 \right)^{1/12} \right] \left( \$70,000.00 \& \$160.00 \right) \\ &= 0.004090797 \left( \$69,840 \right) \\ &= \$285.70 \end{aligned}$$

F. At 120% PSA the prepayment rate is 7.2% instead of 6%

$$\begin{aligned} \text{Rate} &= \text{PSA} \left( 0.06 \left( \min \left( 1, \frac{\text{months}}{30} \right) \right) \right) \\ &= 1.20 \left( 0.06 \left( \min \left( 1, \frac{31}{30} \right) \right) \right) \\ &= 0.072 \end{aligned}$$

and the expected pre-payment is \$433.54 instead of \$359.19

$$\begin{aligned} E[P] &= \left[ 1 + (1 + \text{Prepayment Rate})^{1/12} \right] \left( \text{Outstanding Principal} + \text{Scheduled Principal Payment} \right) \\ &= \left[ 1 + (1 + 0.072)^{1/12} \right] \left( \$70,000.00 + \$160.00 \right) \\ &= (0.006207615) \left( \$69,840 \right) \\ &= \$433.54 \end{aligned}$$



## 20 - 4:

(C) Sallie Mae is the Student Loan Marketing Association.



## 20 - 5:

(B) The Public Securities Association model is a benchmark of prepayment rates for new mortgage loans.